MOS 4.3 – Physical problems in unbounded domains

Activity 2. Numerical resolution of the exterior problem with Freefem++

We consider $\mathbb{D} = \{x \in \mathbb{R}^2 \ ; \ |x| = 1\}$, $\Omega = \mathbb{R}^2 \setminus \overline{\mathbb{D}}$, and the problem

$$\begin{cases} -\Delta u = 0, \text{in } \Omega, \\ u = g, \text{on } \partial\mathbb{D}, \\ u \text{ bounded}. \end{cases} \tag{1}$$

In the following, r and θ denote the polar coordinates centered at the origin. We will assume a particular form for the Dirichlet datum g :

$$g(\mathbf{x}) = a_0 + \cos(\theta) + 2\sin(2\theta) - \cos(3\theta).$$

The exact solution of Problem (1) reads

$$u(\mathbf{x}) = a_0 + \frac{\cos(\theta)}{r} + 2\frac{\sin(2\theta)}{r^2} - \frac{\cos(3\theta)}{r^3}.$$

The goal of this activity is to test the approximate boundary conditions (A.B.C.) on an artificial boundary $\Gamma_R = \{x \in \mathbb{R}^2 \ ; \ |x| = R\}$.

— Neumann A.B.C.

$$\frac{\partial u}{\partial n} = 0 \text{ on } \Gamma_R.$$

— Ventcel A.B.C.

$$\frac{\partial u}{\partial n} - R\frac{\partial^2 u}{\partial \tau^2} = 0 \text{ on } \Gamma_R.$$

1. Download the software `Freefem++` on the webpage

   http://www.freefem.org/

2. Download the sample file `SampleInteriorLaplaceDirichlet.pde` on the webpage

   http://greg.vial.free.fr/Dwl/

   Open it in a text editor and run it with `Freefem++`.

3. Adapt the sample file to solve the problem with the different A.B.C.
   Make tests with different values of $R = 5, 10, 15$.