Performance and robustness analysis of nonlinear closed loop systems using μ_{nl} analysis: Applications to nonlinear PI controllers[1]

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Abstract. Recently, an approach based on the incremental norm was proposed for formalizing nonlinear performance as a well posed optimization framework. Many realistic specifications can be considered in this framework. The underlying philosophy is close to the H_∞ approach proposed by Zames in 1981. In this paper, we illustrate the strong interest of this approach for analyzing nonlinear PID control of linear time invariant (LTI) systems on an example (a nonlinearity is introduced for improving noise filtering). This analysis is based on the extension of the μ analysis (LTI robustness analysis) to nonlinear systems, referred to as μ_{nl} in this paper.

1 Introduction

In many industrial applications, automatic control designers have confronted to nonlinear closed-loop systems. In some cases, even if a LTI model of to-be-controlled process is accurate, the closed loop performance obtained with usual LTI control law can be dramatically improved by introducing a nonlinearity in the control law, see e.g. nonlinear PID controllers. In other cases, the to-be-controlled process can not be assumed to be LTI (see e.g. missiles, ships, irrigation canals, etc.). Nevertheless, LTI control law can ensure desired closed loop performance.

The basic feature of these applications is that the automatic control designer has to design or to improve a controller in order to ensure performance and robustness specifications for the nonlinear closed loop system. The main difficulty is that the specifications are not local and that the input signals are not necessarily constant inputs. For instance, usual specifications can be: steady state behaviors with respect to classes of inputs (steps, periodic signals, etc), rejection of disturbance classes, expressed *e.g.* as a norm ratio between the disturbance and the tracking error, noise effect limitation, robustness with respect to system uncertainties, etc.

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Actually, when the closed loop system can be assumed LTI, closed loop performance specifications can be readily analyzed using *e.g.* classical tools such as the closed loop and open loop frequency responses, pole and zero location or more recent tools as *e.g.* μ-analysis. Unfortunately, such tools are not "available" when analyzing nonlinear systems. As a consequence, the nonlinear closed loop performance is usually "validated" by intensive time domain simulations with few guarantees for the non simulated scenarios. Consequently, there is a strong interest in developing *analysis* methods, allowing performance validation when the closed loop is nonlinear.

In the nonlinear approach, a possible solution to this problem can be found in the incremental norm framework [10, 5]. Remember first that a nonlinear system H is said to be incrementally bounded on $\mathcal{L}_2$, the set of energy bounded signals, if there exists a positive constant η such that $\|H(u_1) - H(u_2)\|_2 \leq \eta\|u_1 - u_2\|_2$ for all u_1 and u_2 in $\mathcal{L}_2$. The incremental norm framework is useful for analyzing both qualitative properties and quantitative of nonlinear closed loop systems, including desensitivity (see [10] for details).

In a qualitative way, incrementally stable systems possess suitable steady-state properties:

1. all the unperturbated trajectories are globally asymptotically Lyapunov stable;
2. for a given input signal, there is a unique steady-state motion independently of the initial condition and despite a vanishing perturbation on the input signal;
3. the steady state response to a constant (resp. periodic) input signal is also constant (resp. periodic).

These properties are usual control specifications. Note that $\mathcal{L}_2$-gain stability of the closed loop system does not ensure these properties [9]. Consequently, this fact emphasizes the special interest of the incremental norm in an engineering context when considering qualitative properties.

Furthermore, in this framework, it is possible to analyze quantitative robustness and performance properties of a nonlinear closed loop system. The weighted incremental norm approach was indeed originally introduced as an extension of the at now classical H_∞ control concepts from the LTI systems to the nonlinear systems [5, 10]. When considering LTI systems, the original idea of [22] was to recast the initial design problem into a well defined optimization problem, involving the minimization of a *weighted* H_∞ norm.

In the same way, the underlying idea of the incremental norm approach is to define the robustness and performance properties of a (nonlinear) system as optimization constraints involving *weighted* incremental norms. The introduction of a weighting function allows to explicitly take into account the desired quantitative properties for the closed loop systems.

As it was pointed out in [5, 9], testing incremental stability using necessary and sufficient conditions is not an easy task since involving the resolution of Hamilton-Jacobi like equations. Nevertheless, based on sufficient conditions, incremental stability can be tested using efficient convex optimization involving Linear Matrix Inequality constraints. This approach could present a certain conservatism. Nevertheless, our main purpose is to achieve a trade-off between this possible conservatism and the complexity of the considered test. Such an approach was adopted in [9] by introducing "nonlinear μ tools". Due to the strong link with the (LTI) μ analysis, such tests are denoted in the sequel μ_{nl}. These tests rely on well-known input/output approach results (see *e.g.* [21], as well as recent results such as [15, 16]). A successful application to the analysis of the a realistic nonlinear missile PI control was presented in [9].

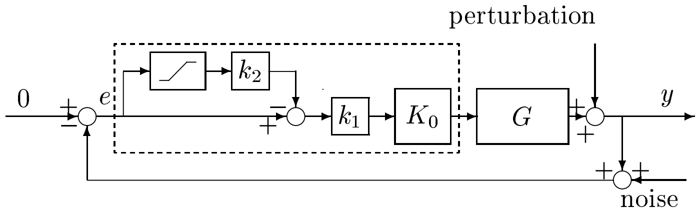
In this paper, we apply these tools to the analysis the *nonlinear* PI-like control of linear time invariant processes. What is the interest of considering such a problem? In this class of controllers, a nonlinearity is introduced in order to improve closed loop system performance with respect to the linear closed loop system. Consequently, the main problem is to test if the performance specifications (*i.e.*, input margin, steady state, perturbation rejection, etc.), ensured in the linear design, are at least recovered in the nonlinear one.

In the sequel, we illustrate how the incremental norm framework application and the use of the μ_{nl}-analysis, allow to prove that the nonlinear PI controller performance recovers (at least) the linear PI controller performance. An extended version of this paper is available as a technical report [12].

2 Considered case: Nonlinear filtering

Engineers use to improve the performance of classical PI(D) based controllers by introducing nonlinearities in the controller, in a heuristic way. Numerous approaches are possible; the most well known approach is probably the fuzzy approach (see e.g., [20]). The main difficulty is then to ensure that some performance specifications are satisfied with the nonlinear controller.

Our purpose is to discuss and to emphasize the interest of μ_{nl} analysis for analyzing such controllers. Of course, considering related applications, there is numerous classes of design specifications for controlling different classes of systems. For the sake of clarity, we choose to focus on a particular problem: PI(D) control with nonlinear filtering of a low order plant. Note that our approach is not specifically dedicated to these problems.



To-be-controlled plant we consider a second order one:

$$G(s) = \frac{K}{(\tau_1 s + 1)(\tau_2 s + 1)}$$

with $K = 9.09$, $\tau_1 = 0.1\,s$ and $\tau_2 = 2 \times 10^{-2}\,s$.

Specifications The control specification is to reject output perturbation despite important measurement noise (high noise/signal ratio at low frequency) with a limited noise magnitude.

Controller design When designing linear time invariant controller, it is a well-known fact that there is a trade-off between rejection time of the output perturbation and noise effect attenuation, especially on control input. In many situations, the noise/signal ratio is low at low frequency allowing a large control bandwidth thus a small rejection time.

In the considered case, the noise/signal ratio is high at

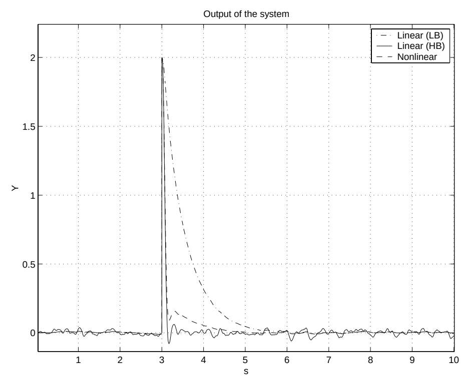


Figure 1: System output with low gain controller (dash-dotted line), high gain controller (full line) and nonlinear controller (dash line)

low frequencies, but the noise magnitude is limited. The controller is then modified using the classical idea which is to introduce a nonlinearity such that controller gain is decreased when the tracking error magnitude gets below a certain level. Most precisely, let us denote e the tracking error and u the command input. Then, the controller has the following structure:

$$u(s) = kK_0(s)e(s)$$

with k a constant gain and $K_0(s) = \frac{\tau_z s+1}{s(\tau_c s+1)^2}$ where $\tau_z = \frac{1}{8}$, $\tau_c = \frac{1}{100}$.

If the constant k is "small", that is $k = k_{min} = 0.2$ then the noise effect is attenuated on the command input and on the system output but the output perturbation rejection is slow (see figure 1 and figure 2). If k is "big", that is $k = k_{max} = 1.45$ then the output perturbation rejection is fast but the noise effect is not attenuated on the command input and on the system output (see figure 1 and figure 2). Note that the difference between two controller bandwidths is around one decade.

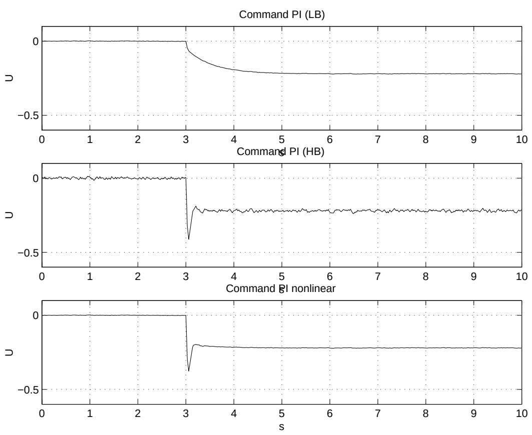


Figure 2: Command input u with low gain controller (top), high gain controller (center) and nonlinear controller (bottom)

A nonlinear controller can be obtained by scheduling the constant k as a function of the tracking error e:

$$k(e(t))e(t) = k_1 e(t) - k_2 \mathbf{sat}(e(t))$$

where $k_1 = k_{max}$, $k_2 = k_{min}/k_{max} - 1$ and the saturation function **sat** is defined as follows:

$$\begin{array}{rclcrcl} \mathbf{sat}(e(t)) & = & -0.2 & e(t) & \leq & -0.2 \\ \mathbf{sat}(e(t)) & = & e(t) & |e(t)| & \leq & 0.2 \\ \mathbf{sat}(e(t)) & = & 0.2 & e(t) & \geq & 0.2. \end{array}$$

From figure 1 and figure 2 note that controller structure allows to get both the benefit of the low gain controller and the benefit of the high gain controller: a good noise attenuation with a good rejection time.

3 Incremental norm approach (see [12])

Notations $\mathcal{L}_2$ is the space of $\mathcal{R}^n$ valued functions defined on $\mathcal{R}$, where the norm is defined by $\|f\|_2 = (\int \|f(t)\|^2 dt)^{1/2}$. $P_T f$ is defined by $P_T f(t) = f(t)$ for $t \leq T$ and 0 otherwise. The *extended space* $\mathcal{L}_2^e$ is the space of $\mathcal{R}^n$ valued functions defined on $\mathcal{R}$ whose causal truncations belong to $\mathcal{L}_2$. Let us consider a nonlinear system, $y = \Sigma_{x_0}(u)$:

$$\Sigma_{x_0} \left\{ \begin{array}{rcl} \dot{x}(t) & = & f(x(t), u(t)); \quad x(t_0) = x_0, \\ y(t) & = & h(x(t), u(t)) \end{array} \right. \qquad (1)$$

where $x(t) \in \mathcal{R}^n, u(t) \in \mathcal{R}^p$ and $y(t) \in \mathcal{R}^m$. f and h are assumed uniformly Lipschitz and C^1 with $f(x_0, 0) = 0$ and $h(x_0, 0) = 0$.

Definition Σ_{x_0} has a finite incremental gain on $\mathcal{L}_2$ if there exists $\eta \geq 0$ such that

$$\|\Sigma_{x_0}(u_1) - \Sigma_{x_0}(u_2)\|_2 \leq \eta \|u_1 - u_2\|_2$$

for all $u_1, u_2 \in \mathcal{L}_2$. The incremental gain of Σ_{x_0}, denoted $\|\Sigma_{x_0}\|_\Delta$, is the minimum value of η. Σ_{x_0} is said to be incrementally stable if it is stable, *i.e.* it maps $\mathcal{L}_2$ to $\mathcal{L}_2$, and has a finite incremental gain.

A sufficient condition for the incremental stability was proposed in [9], involving solving convex optimizations problem on Linear Matrix Inequality (LMI) constraints[1], for which efficient numerical algorithms have been proposed [1].

Theorem 3.1 *Σ_{x_0}, defined by (1), has an incremental gain less or equal to η for any initial condition $x_0 \in \mathcal{R}^n$ if there exists a symmetric and positive definite matrix P such that for all $x \in \mathcal{R}^n$ and $u \in \mathcal{R}^p$ the following matrices are definite positive:*

$$\begin{bmatrix} P\,\frac{\partial f}{\partial x}(x,u) + \frac{\partial f}{\partial x}(x,u)^T\,P & P\,\frac{\partial f}{\partial u}(x,u) & \eta^{-1}\frac{\partial h}{\partial x}(x,u)^T \\ \frac{\partial f}{\partial u}(x,u)^T\,P & -I_{p\times p} & \eta^{-1}\frac{\partial h}{\partial u}(x,u)^T \\ \eta^{-1}\frac{\partial h}{\partial x}(x,u) & \eta^{-1}\frac{\partial h}{\partial u}(x,u) & I_{m\times m} \end{bmatrix} \qquad (2)$$

Note that the previous condition can be satisfied only if there exists a symmetric and positive definite matrix P such the following condition: for all $x \in \mathcal{R}^n$, for all $u \in \mathcal{R}^p$,

$$P\,\frac{\partial f}{\partial x}(x,u) + \frac{\partial f}{\partial x}(x,u)^T\,P < 0. \qquad (3)$$

This condition ensures that there exists a finite $\eta > 0$ such that the operator which links the input of system (1) to its state is incrementally stable *i.e.* $\|x_1 - x_2\|_2 \leq \eta \|u_1 - u_2\|_2$ where x_1 and x_2 are respectively the solution of system (1) for the input u_1 and u_2.

We now point out that some robustness and performance problems of multi-input multi-output closed loop nonlinear systems can be formulated as well posed optimization problems, in the same way as in the linear context [22, 3]. For a complete explanation see [5, 9]. Due to the considered exemple we mainly focus on perturbation rejection.

[1]The underlying idea is to compute a particular solution of the Hamilton Jacobi like equation.

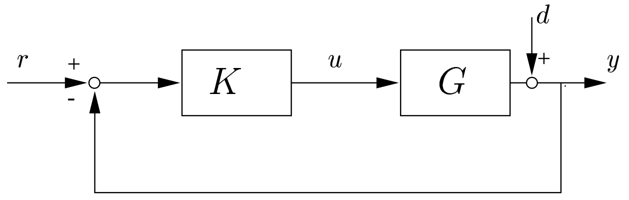


Figure 3: The perturbed closed-loop system

Perturbation specification The use of feedback control (see figure 3) is mainly linked to their ability to reduce the effect of non measurable perturbations or to shrink model uncertainties: the desensitivity property [2, 22]. We introduce a set of possible perturbations, namely $P^e \subset \mathcal{L}_2^e$. As in the H_∞ approach, we now assume that the set of possible perturbations P^e can be defined by:

$$\{d \in \mathcal{L}_2^e | \|W_p^{-1}(d) - W_p^{-1}(r+d)\| \leq \epsilon \|d\| \text{ for any } r \in \mathcal{L}_2^e\}$$

where W_p and W_p^{-1} are two causal and incrementally stable operators. By following [2, 10], comparison desensitivity to the considered perturbation is said achieved if the "weighted" incremental norm of the sensitivity function $S = (I + GK)^{-1}$ is less than 1, that is: $\|SW_p\|_\Delta \leq 1$.

Robustness against unstructured uncertainties We consider the case of unstructured uncertainties $\widetilde{\Delta}$,on the plant model G. This model perturbation $\widetilde{\Delta}$ may represent uncertainties on the actuator or sensor dynamics, and more generally neglected dynamics. $\widetilde{\Delta}$ is assumed belonging to $\Omega_{\widetilde{\Delta}} = \left\{ \widetilde{\Delta} = W_3 \Delta W_2 \mid \|\Delta\|_\Delta < 1 \right\}$ where Δ is a (possibly nonlinear) causal operator from $\mathcal{L}_2^e$ to $\mathcal{L}_2^e$ and W_2 and W_3 are known, causal, incrementally stable input-output maps from $\mathcal{L}_2^e$ to $\mathcal{L}_2^e$. A stability result is proposed for the interconnected system of figure 4.

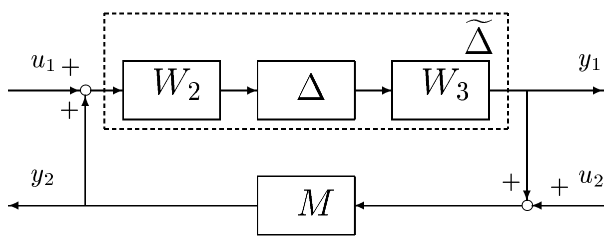


Figure 4: Robustness analysis of nonlinear feedback

If M is incrementally stable and if the following inequality holds:

$$\|W_2 M W_3\|_\Delta \leq 1$$

then the closed loop system of figure 4 is incrementally stable for any uncertainty $\widetilde{\Delta}$ belonging to $\Omega_{\widetilde{\Delta}}$.

Performance with uncertainties The performance of the closed system of figure 5 is incrementally robust if it holds for every $\widetilde{\Delta} \in \Omega_{\widetilde{\Delta}}$, that is:

$$\forall \widetilde{\Delta} \in \Omega_{\widetilde{\Delta}}, \quad \|(I + FG(I - \widetilde{\Delta})^{-1}K)^{-1}W_1\|_\Delta \leq 1$$

As in the linear case, conditions ensuring performance

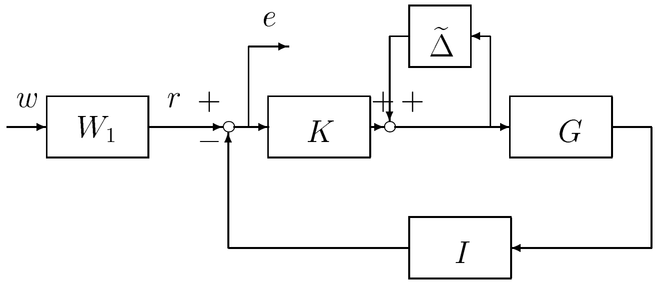


Figure 5: Robust incremental performance

against a perturbation and incremental robustness can be combined to capture both performance and uncertainties in a single statement.

4 μ_{nl}-analysis for practical analysis of incremental properties

We now propose computationally attractive conditions ensuring robust *incremental* stability and robust *incremental* performance. The closed loop nonlinear system is rewritten as the connection of a stable LTI system $M(s)$ with an operator Δ, which simultaneously contains the uncertainties and the nonlinearities:

$$\left\{ \begin{array}{ccc} p & = & \Delta(q) \\ \left[\begin{array}{c} q \\ e \end{array} \right] & = & M\left(\left[\begin{array}{c} p \\ w \end{array} \right] \right) \end{array} \right. \quad \text{and} \quad M = \left[\begin{array}{cc} M_{qp} & M_{qw} \\ M_{ep} & M_{ew} \end{array} \right] \tag{4}$$

where e is the output of the system and w its input. p and q are internal signals. The operator Δ, which thus models the uncertain and nonlinear part of the system, has the following structure:

$$\Delta \stackrel{\Delta}{=} \mathbf{diag}\,(\Delta_1, \ldots, \Delta_i, \ldots, \Delta_r)\,. \tag{5}$$

Each sub-block of Δ belongs to a certain class of uncertain and nonlinear systems.

- Δ_i may be an (unstructured) model uncertainty: it is thus a multi-input multi-output operator, which is only known to be incrementally passive[2]; depending on the considered problem, Δ_i can be nonlinear time variant, linear time invariant or even a uncertain parameter;
- Δ_i may be otherwise a single input, single output

[2] In the previous section, we consider uncertainties whose incremental gain was bounded by 1. Note that using loop shifting transformation, the interconnected system can be readily rewritten as an interconnection with a incrementally passive incertainties.

static nonlinearity, *i.e.* $\Delta_i = f_i$, where f_i is an incrementally passive nonlinearity.

One way to prove the (robust) incremental stability of the connection of M_{qp} with Δ is to directly apply the incremental passivity theorem [21]. Nevertheless, such an approach can lead to a conservative criterion. One way to reduce this conservatism is to introduce *multipliers* [21, 15, 16].The basic idea is to transform the connection of M_{qp} with Δ into a new connection of $\widetilde{M}_{qp}$ with $\widetilde{\Delta}$, which has the same stability properties, i.e. the former is stable if and only if the latter is also stable. Furthermore, if Δ is incrementally passive, then $\widetilde{\Delta}$ must also be chosen incrementally passive.

Families of multipliers are now proposed for each Δ_i sub-block class. Assume first that Δ_i is a passive multi-input multi-output nonlinear time variant model uncertainty or a static nonlinearity. Then the operator $m\Delta$ is also passive for any real $m > 0$[3]. m is thus a *multiplier*. In addition, if Δ_i is linear time invariant, then m can be taken as frequency dependent, with no pole or zero on the imaginary axis [15]. When considering the overall operator Δ of equation (5), let us now introduce the set of multipliers $\mathcal{M}_{mul}(\Delta)$ with the following structure: $M_{mul} \triangleq \mathbf{diag}\,(m_1 I, \ldots, m_i I, \ldots, m_r I)$ where m_i is possibly frequency dependent if Δ_i is linear time invariant or an uncertain parameter.

Theorem 4.1 *The connection of M_{qp} with Δ is incrementally stable, for any Δ which is incrementally passive, if there exists a multiplier $M_{mul} \in \mathcal{M}_{mul}(\Delta)$ satisfying:*

$$\text{for all } \omega, \quad M_{mul}(j\omega)M_{qp}(j\omega) + (M_{mul}(j\omega)M_{qp}(j\omega))^* < 0. \tag{6}$$

Theorem (4.1) can be applied to prove the robust incremental stability property.

We now focus on the (robust) *incremental performance* evaluation. We propose conditions for ensuring that the interconnected system (4) has an incremental gain less than one. Using a loop-shifting transformation , this problem can be transformed in ensuring that the connection of $\overline{M}$ with Δ, where $\overline{M}$ is:

$$\begin{bmatrix} M_{qp} + M_{qw}(I - M_{ew})^{-1}M_{ep} & -2M_{qw}(I - M_{ew})^{-1} \\ (I - M_{ew})^{-1}M_{ep} & -I - 2M_{ew}(I - M_{ew})^{-1} \end{bmatrix}$$

is incrementally passive. To this purpose, let us introduce the set of multipliers $\mathcal{M}_{mul}^{perf}(\Delta)$ of the form: $M_{mul}^{perf} \triangleq \mathbf{diag}\,(M_{mul}, m_{perf} I)$ where $M_{mul} \in \mathcal{M}_{mul}(\Delta)$ and m_{perf} is a positive real scalar.

Theorem 4.2 *System (4) is incrementally passive, for any Δ which is incrementally passive, if there exists a multiplier $M_{mul}^{perf} \in \mathcal{M}_{mul}^{perf}(\Delta)$ satisfying:*

$$\text{for all } \omega, \quad M_{mul}^{perf}(j\omega)\overline{M}(j\omega) + \left(M_{mul}^{perf}(j\omega)\overline{M}(j\omega)\right)^* < 0. \tag{7}$$

[3]Kulkarni and Safonov in [13] prove that in the case of nonlinear operator, only constant multipliers are possible.

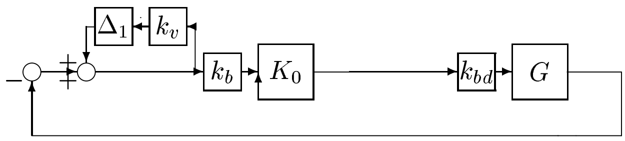


Figure 6: Closed loop for incremental stability analysis

Testing condition (6) and testing condition (7) boil down to convex optimization problems which are infinite dimensional. The optimization variable is M_{mul} (or M_{mul}^{perf}) that is an infinite dimensional variable.

In order to obtain an attractive test, we are interested by a suffisant condition which is a finite dimensional convex optimization problem. First, in condition (6), the multipliers M_{mul} is researched in a state space representation: $M_{mul}(s) = D_{mul} + C_{mul}(sI - A_{mul})^{-1}B_{mul}$. In order to get a finite dimensional variable, the matrices A_{mul} and B_{mul} are fixed. M_{mul}^{perf} can be similarly discussed. Furthermore, remember that, given a real rational transfer function $H(j\omega) = D + C(j\omega I - A)^{-1}B$, Kalman Yakubovich Popov (KYP) lemma [17] claims that testing that $\forall\omega$, $H(j\omega) + H(j\omega)^* > 0$ is equivalent to finding a symmetric matrix P such that :

$$\begin{bmatrix} A^T P + PA & PB + C \\ B^T P + C^T & D + D^T \end{bmatrix} > 0.$$

Actually, this classical lemma allows to replace an infinite number of inequalities by one (matrix) inequality by introducing an additional decision variable P. By applying this lemma, testing (6) and testing condition (7) reduce to finite dimensional convex optimization involving Linear Matrix Inequality constraints where the optimization variables are P, D_{mul} and C_{mul}. A similar approach was adopted in [15].

Remarks: Note the strong link between conditions (6) and (7) and the μ upper bound (in the form presented in [15]) for the robustness analysis of LTI systems. The basic difference is in the considered classes of multipliers M_{mul}. When analyzing LTI systems, for two different frequencies ω_1 and ω_2, $M_{mul}(j\omega_1)$ can be computed independently of $M_{mul}(j\omega_2)$. It is no longer true when *nonlinear* systems are considered. In our problems, $M_{mul}(j\omega_1)$ explicitly dependent on $M_{mul}(j\omega_2)$. As a consequence, μ analysis based on frequency gridding cannot be applied when considering nonlinear systems.

5 Application of the $\mu_n l$ tool to PI control with nonlinear filtering

Let us introduce $k_v = 1 - \frac{k_b}{k_{max}}$ and $k_b = \frac{2}{\frac{1}{k_{min}} + \frac{1}{k_{max}}}$ $k_{bd} = \frac{2}{1 + \frac{1}{\Delta G^u}}$ and $k_{vd} = 1 - \frac{k_{bd}}{\Delta G^u}$.

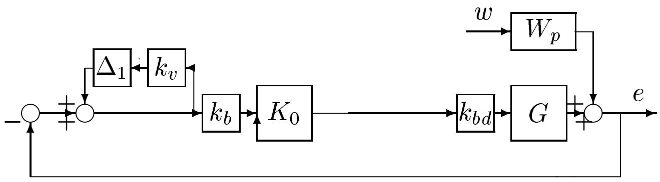


Figure 7: Incremental performance analysis

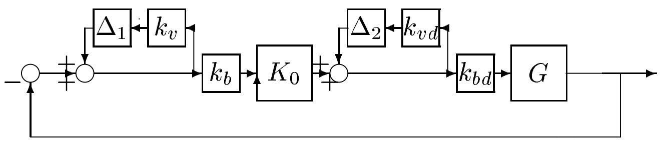


Figure 8: Robust incremental stability analysis

Incremental stability Incremental stability is analyzed by testing condition (6) of Theorem 4.1. M is defined from the closed loop system represented figure 6 where Δ_1 is a nonlinear operator which corresponds to the PID nonlinearity. Condition (6) is satisfied with the multiplier $W_{mul} = 0.103$.

Incremental performance Incremental performance is analyzed by testing condition (7) of Theorem 4.2. M is defined from the closed loop system represented figure 7 where $W_p(s)$ is weighting function defined as: $W_p(s) = 0.556\frac{10^{-4}s+4.4}{s+10^{-4}}$. This weighting function ensures fast step perturbation rejection (rejection bandwidth greater than $2.2\,rad/s$). Condition (7) is satisfied with the multiplier $M_{mul}^{perf} = diag(3.58, 1.56)$.

Robust incremental stability We now investigate incremental stability with respect to a DC gain uncertainty. See the closed loop system presented in figure 8 where Δ_2 is an parametric uncertainty which correspond to an upper gain margin. Let us denote ΔG^u the guaranteed lower bound on the upper gain margin. Then, with $\Delta G^u = 7.5\,dB$, condition (6) of Theorem 4.1 is satisfied with the multiplier $M_{mul}(j\omega) = diag(1.28, 2.34, \frac{(j\omega-29.62)(j\omega+0.68)}{(j\omega+1)(j\omega-1)})$

Robust incremental performance We now investigate incremental performance with DC gain uncertainty (see figure 9). To this purpose, the performance is relaxed with respect to performance analysis without DC gain uncertainty: $W_p(s) = 0.4\frac{10^{-4}s+4.4}{s+10^{-4}}$ (rejection bandwidth greater than $1.7\,rad/s$). As we investigate performance, the gain margin is reduced: $\Delta G^u = 6dB$. Condition (7) is then satisfied with the multiplier: $M_{mul}^{perf}(j\omega) = diag(1.1, 2.62\frac{(j\omega-36.4)(j\omega-6.4)(j\omega+8.9)(j\omega+0.9)}{(j\omega+10)(j\omega-10)(j\omega+1)(j\omega-1)}, 0.4)$.

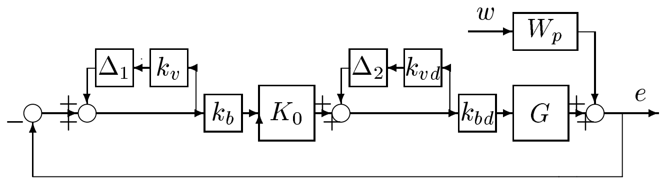


Figure 9: Robust incremental performance analysis

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