First Test - Mathematics - BSc

30/08/24

Functions, graphs and integrals

1. In a cartesian coordinates system:
   (a) draw the graph of the function $x \longmapsto |x| - 1$,
   (b) deduce the graph of $x \longmapsto ||x| - 1|$.
2. About the parabola $x \longmapsto x^2$:
   (a) Draw the set
   $$E = \{(x, y) \in \mathbb{R}^2 /\ 0 \leq x \leq 1 \text{ and } 0 \leq y \leq x^2\}.$$
   (b) Calculate the area of E.
3. Let f be defined by
   $$f(x) = \frac{1}{x^2} \text{ for } x \neq 0.$$
   (a) Draw the graph of f.
   (b) What is the image of f, $f(\mathbb{R})$?
   (c) What is the inverse image of $(-1, 1]$ by $f^{-1}((-1, 1])$?
   (d) Give the inverse of f, f^{-1}, from $f(\mathbb{R})$ to $(0, +\infty)$.

Sequences and induction

4. Let $(u_n)_{n\geq 0}$ be the sequence defined by
   $$\begin{cases} u_{n+1} = u_n^2 \text{ for all } n \geq 0 \\ u_0 \in \mathbb{R}. \end{cases}$$
   (a) Give the explicit expression of u_n (in terms of n and u_0) for $n \geq 0$.
   (b) For which value(s) of u_0 does this sequence converges ? An in this case, what is the limit of the sequence. No proof is required.

Complex numbers

5. Help you with a sketch (*FR : dessin*) to solve this problem.
   (a) What is the argument of the complex number $z = 1 + i$?
   (b) Write the polar form (modulus and argument il you do not know what a polar form is) of z^n for $n \geq 0$ (general formula with n in it).
   (c) For which $n \in \mathbb{N}$ do we have $z^n \in \mathbb{R}_+$?

Sets

6. Draw the following subsets of $\mathbb{R}^2$:

$$E = \{(x, y) \in \mathbb{R}^2 /\ \max(|x|, |y|) \leq 1\}.$$

7. What are these sets ?
   (a) $E = \bigcup_{n \in \mathbb{N}} [0, n]$
   (b) $F = \bigcap_{n \in \mathbb{N}} [n, +\infty)$.

Functions and relations

8. We want to define order relations on $\mathbb{R}^2$. Are the following binary relations order relations? If so are they total or partial ? Is one of them similar to the alphabetical order?
   (a) We say that $(x_1, x_2)\ \mathcal{R}\ (y_1, y_2)$ if $x_1 < y_1$ or if $x_1 = y_1$ and $x_2 \leq y_2$.
   (b) We say that $(x_1, x_2)\ \mathcal{R}'\ (y_1, y_2)$ if $x_1 \leq y_1$ and $x_2 \leq y_2$.