M1 Core course in computing for engineers

Exercise sheet 1 - Solving systems

- École Centrale de Lyon -

Exercise 1.

1. Create a 3×3 symmetric matrix A which has a 6 on its diagonal, a -4 on its second diagonal and a 1 at each corner.
2. Modify the `gaussElimin` function in order that it takes into account several constant vectors `b`.
3. Try your new function with matrix A and constant vectors

$$\begin{pmatrix} -14 \\ 36 \\ 6 \end{pmatrix} \text{ and } \begin{pmatrix} 22 \\ -18 \\ 7 \end{pmatrix}.$$

Exercise 2.

1. Write a function `LUdecomp` which takes a square matrix as argument and returns the decomposed matrix in Doolittle's procedure.
2. Write a function `LUsolve` which, from a LU Doolittle's decomposition matrix and a set of constant vectors stored in a matrix B, gives the set of solutions of $LU = B$.
3. Applied both functions in order to solve the system

$$\begin{cases} x_1 + 4x_2 + x_3 = 7 \\ x_1 + 6x_2 - x_3 = 13 \\ 2x_1 - x_2 + 2x_3 = 5. \end{cases}.$$

Exercise 3. Choleski's decomposition can be found in both `numpy.linalg` and `scipy.linalg`. Find out the function, how to use it. Write a function `Choleskisolve` which solves, by using Choleski's decomposition,the system associated to the following data that you may copy-paste:

```
a = np.array([[ 1.44, -0.36, 5.52, 0.0], [-0.36, 10.33, -7.78, 0.0], [ 5.52, -7.78, 28.40, 9.0], [ 0.0, 0.0, 9.0, 61.0]])
b = np.array([0.04, -2.15, 0.0, 0.88])
```

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Exercise 4. Write an adaptation of the Doolittle's procedure to solve tridiagonal systems. Apply it to the system with coefficients

$$A = \begin{pmatrix} 6 & 2 & 0 & 0 & 0 \\ -1 & 7 & 2 & 0 & 0 \\ 0 & -2 & 8 & 2 & 0 \\ 0 & 0 & 3 & 7 & -2 \\ 0 & 0 & 0 & 3 & 5 \end{pmatrix}$$

and constants

$$b = \begin{pmatrix} 2 \\ -3 \\ 4 \\ -3 \\ 1 \end{pmatrix}$$

Exercise 5. Adapt the Doolittle procedure so that it can handle systems which should be pivoted, by using the swap of column described in the lecture. Apply it to

$$A = \begin{pmatrix} 0 & -1 & 1 \\ -1 & 2 & -1 \\ 2 & -1 & 0 \end{pmatrix} \quad b = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$