Lower limb locomotion activity analysis using Triplet Markov Model

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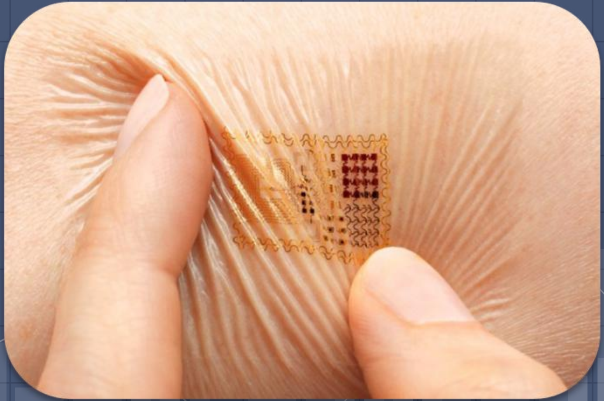
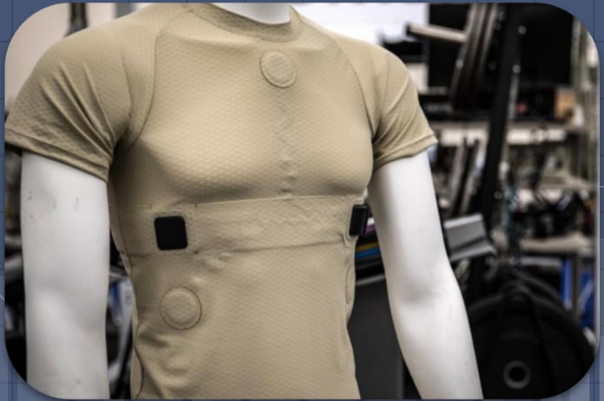
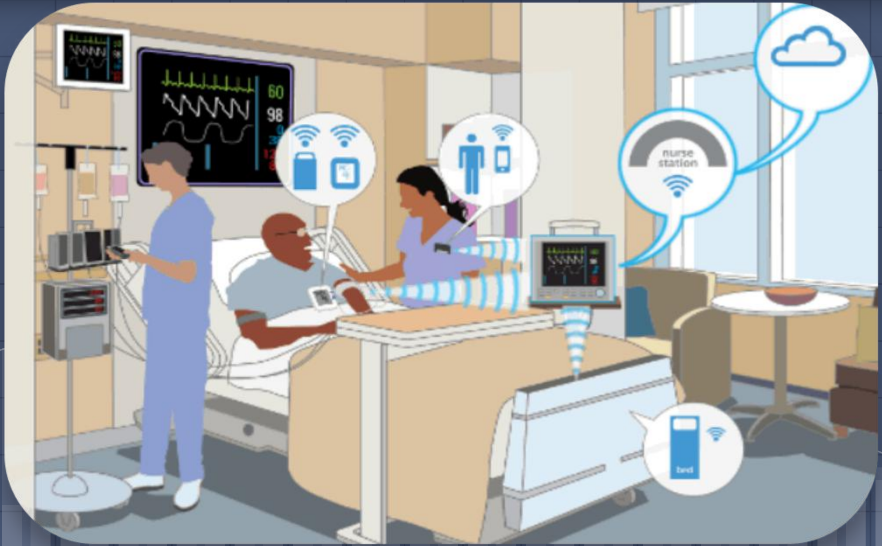
19/11/2019


Current
2:26
Heart Rate
75 BPM

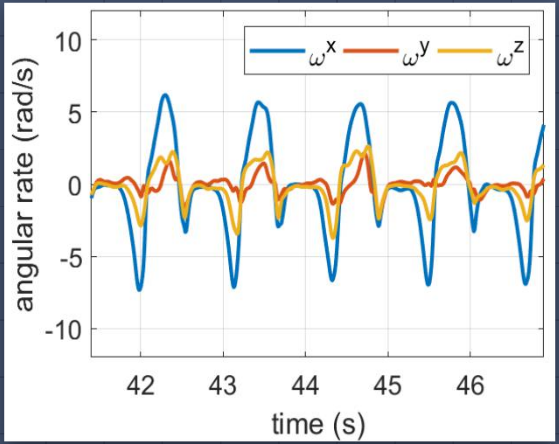


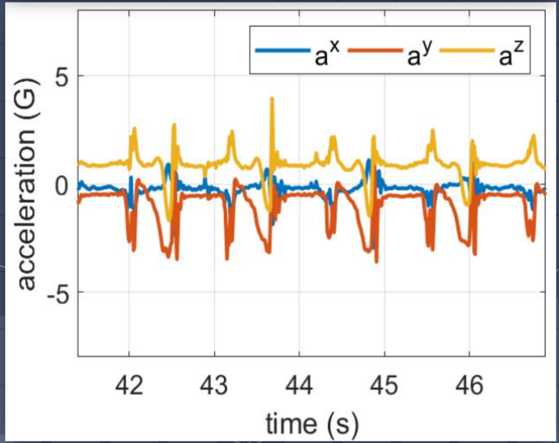

gest




60
98
nurse
station
bed

IMU Sensors & Data

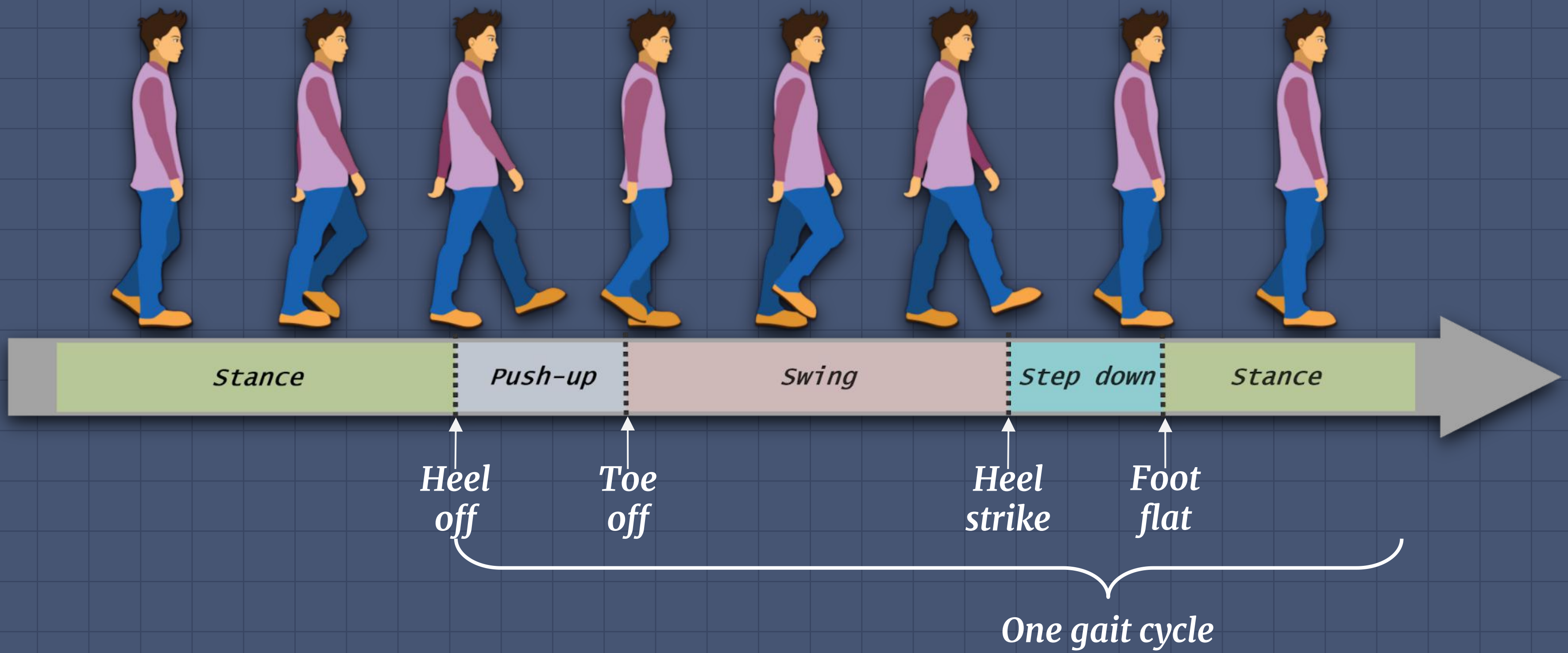




Signals of walking

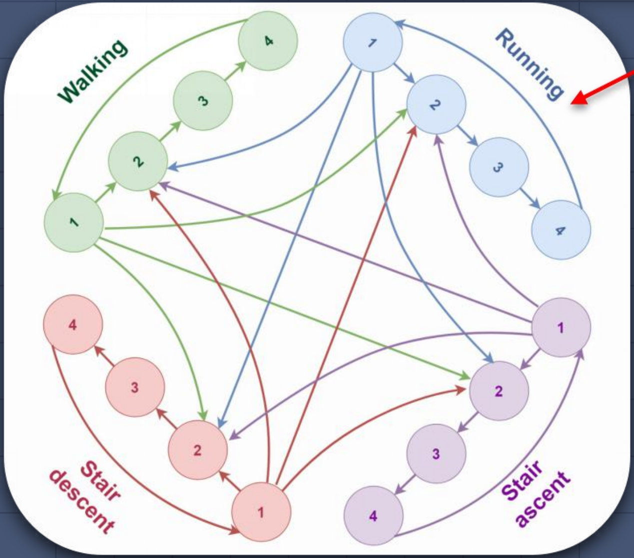
Guideline

1. Gait phases
2. TMC definition for lower limb locomotion activity.
3. Non-parametric TMC
4. Semi Markov structure


Stance
Push-up
Swing
Step down
Stance
Heel off
Toe off
Heel strike
Foot flat
One gait cycle

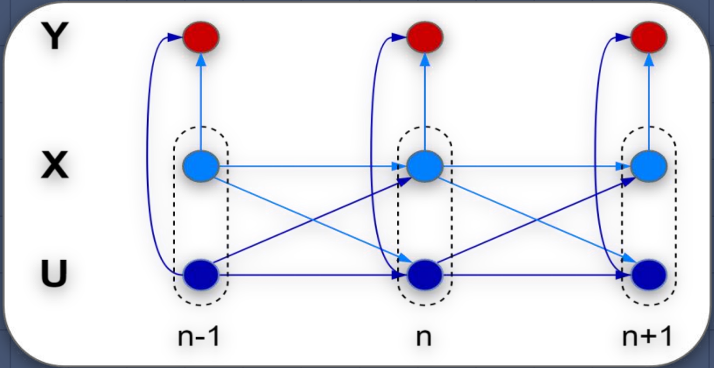
- *U ⟶ Hidden state (Gait phase)*   *X ⟶ Hidden state (Activity)*   *V=(X,U)*
  T=(V,Y) ⟶ TMC
- *The transition of T is given by:*

$$p\left(t_{n+1} \middle| t_n\right) = p\left(v_{n+1}, y_{n+1} \middle| v_n, y_n\right)$$
$$= \underline{p\left(x_{n+1}, u_{n+1} \middle| x_n, u_n\right)}\, \underline{p\left(y_{n+1} \middle| x_{n+1}, u_{n+1}\right)}$$



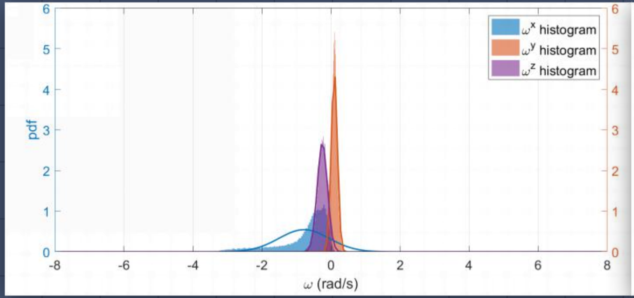
1: Stance; 2: Pushup; 3: Swing; 4: Step down

Gaussian
Histogram
GMM ...

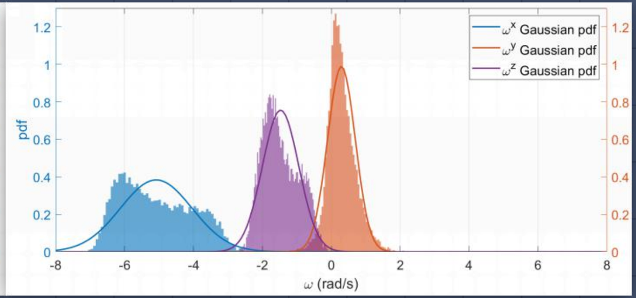


Non-parametric densities:

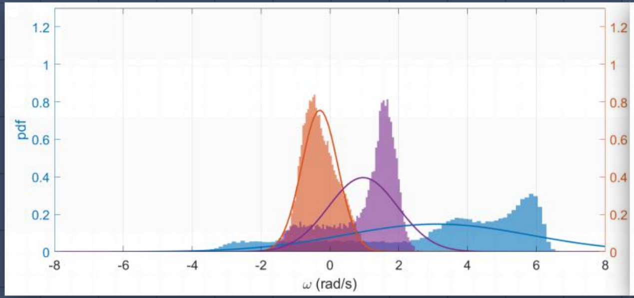
Using classic Gaussian density to represent $p(y_n|x_n,u_n)$ is not very appropriate. So histograms are used to represent the class conditional observation density.



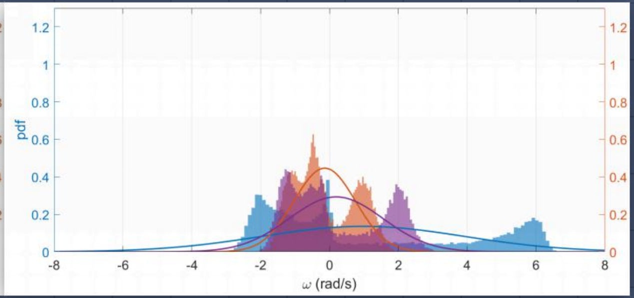
Walking, Stance



Walking, Swing



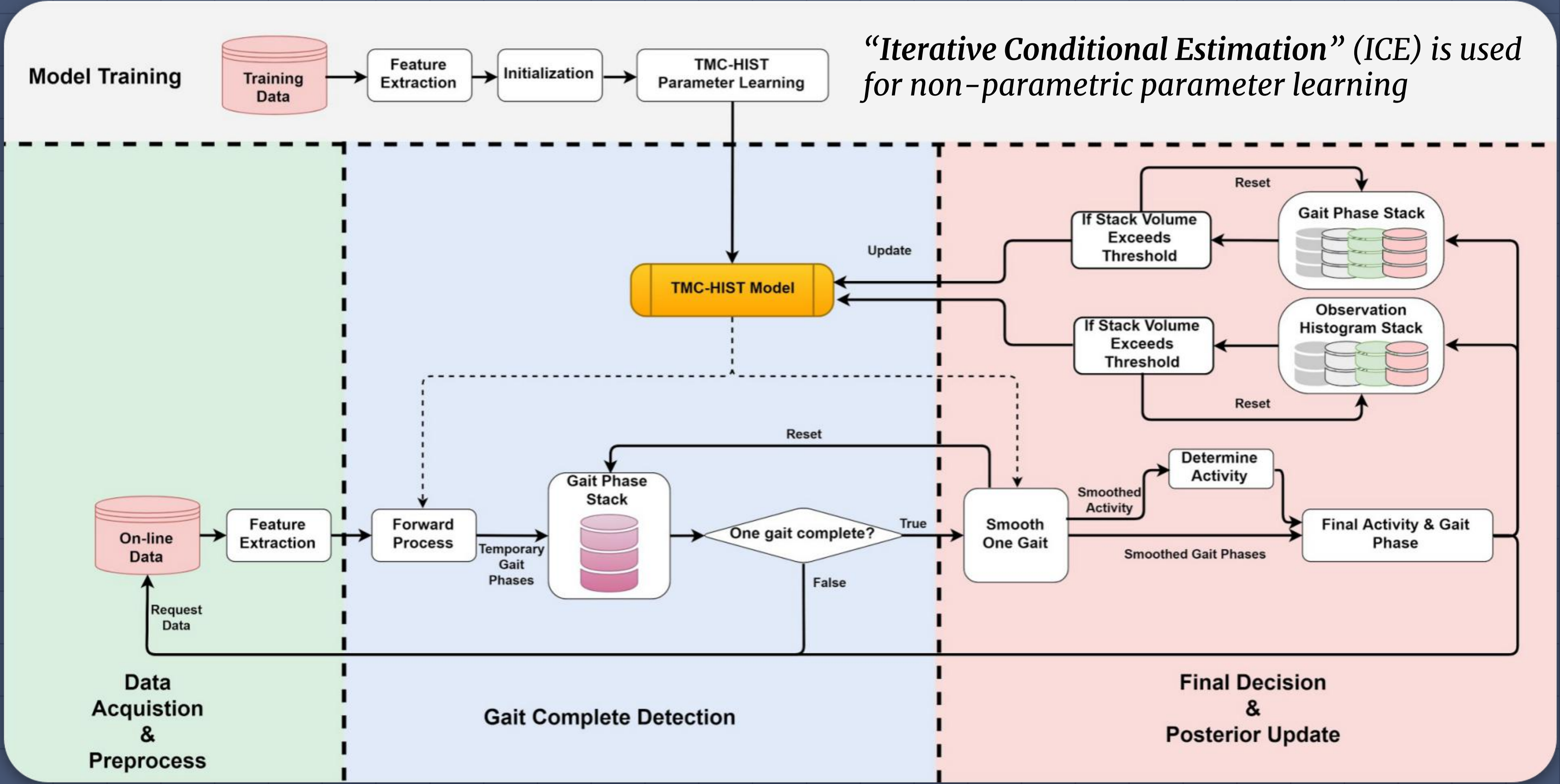
Walking, Push-up



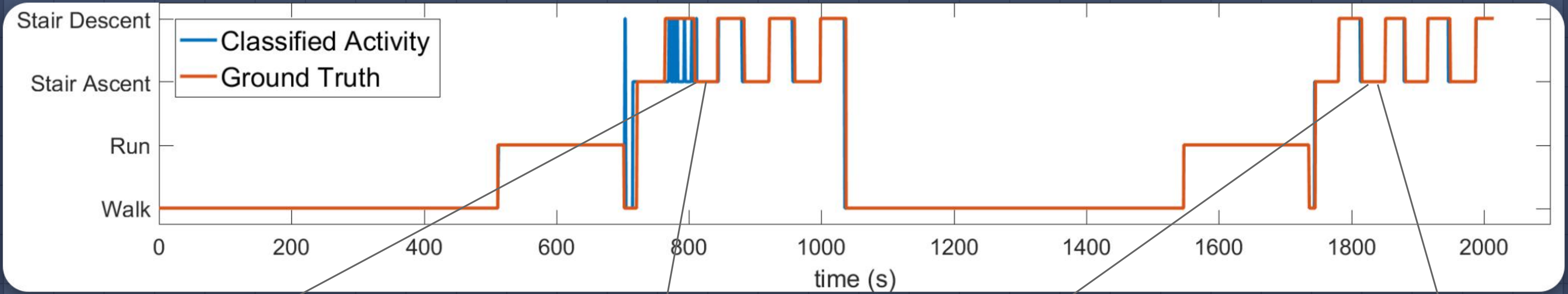
Walking, Step down

On-line recognition

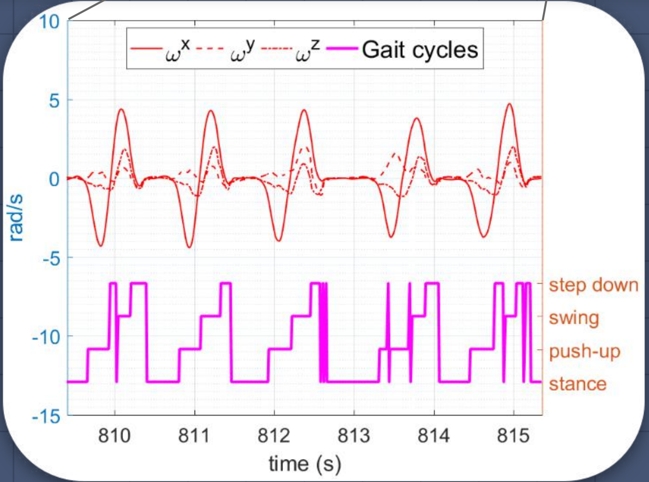
"Iterative Conditional Estimation" *(ICE) is used for non-parametric parameter learning*

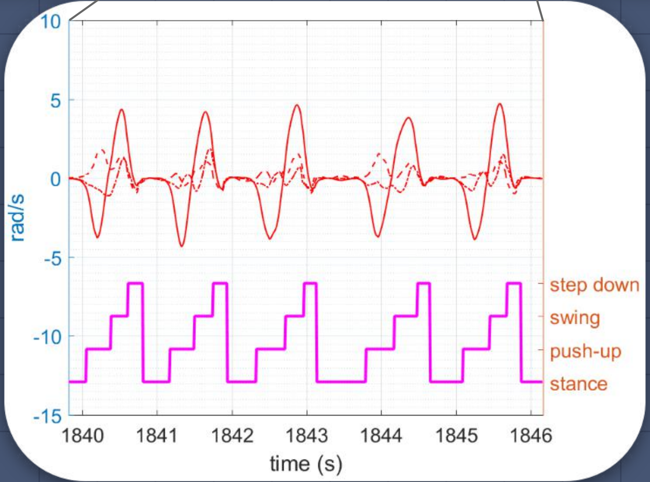


The recognized sequenced activities.



Detected gaits





Limitations of TMC-HIST

1. *The histograms in TMC-HIST does not take the correlations among the three axes into consideration.*
2. *The remaining sojourn time of the hidden states in TMC-HIST is in geometric distribution, whereas it is not very appropriate to the realistic.*

TMC using GMM (TMC-GMM)

- *H ⟶ Gaussian mixture component* *Z=(T,H) ⟶ TMC-GMM*
- *The transition of Z follows:*

$$p\left(\boldsymbol{z}_{n+1} \middle| \boldsymbol{z}_n\right) = p\left(\boldsymbol{v}_{n+1} \middle| \boldsymbol{v}_n\right) p\left(h_{n+1} \middle| \boldsymbol{v}_{n+1}\right) p\left(\boldsymbol{y}_{n+1} \middle| \boldsymbol{v}_{n+1}, h_{n+1}\right)$$

$$p\left(\boldsymbol{y}_n \middle| \boldsymbol{v}_n\right) = \sum_{j=1}^{K} p\left(h_n = j \middle| \boldsymbol{v}_n = i\right) \cdot p\left(\boldsymbol{y}_n \middle| \boldsymbol{v}_n = i, h_n = j\right)$$

$$p\left(h_n = j \middle| v_n = i\right) = c_{ij}$$

Gaussian mixture weights

$$p\left(\boldsymbol{y}_n \middle| \boldsymbol{v}_n = i, h_n = j\right) \sim \mathcal{N}\left(\boldsymbol{\mu}_{ij}, \Sigma_{ij}\right)$$

Mixture component density

TMC-GMM with semi Markov structure (SemiTMC-GMM)

- *D ⟶ Minimum sojourn time* *(Z,D) ⟶ SemiTMC-GMM*
- *The transition of (Z,D) of the proposed SemiTMC-GMM is given:*

$$p(z_{n+1}, d_{n+1} | z_n, d_n) = \underline{p(v_{n+1} | z_n, d_n)}\,\underline{p(d_{n+1} | v_{n+1}, d_n)}\,\underline{p(h_{n+1} | v_{n+1})\, p(y_{n+1} | v_{n+1}, h_{n+1})}$$

Class conditional observation density in TMC-GMM

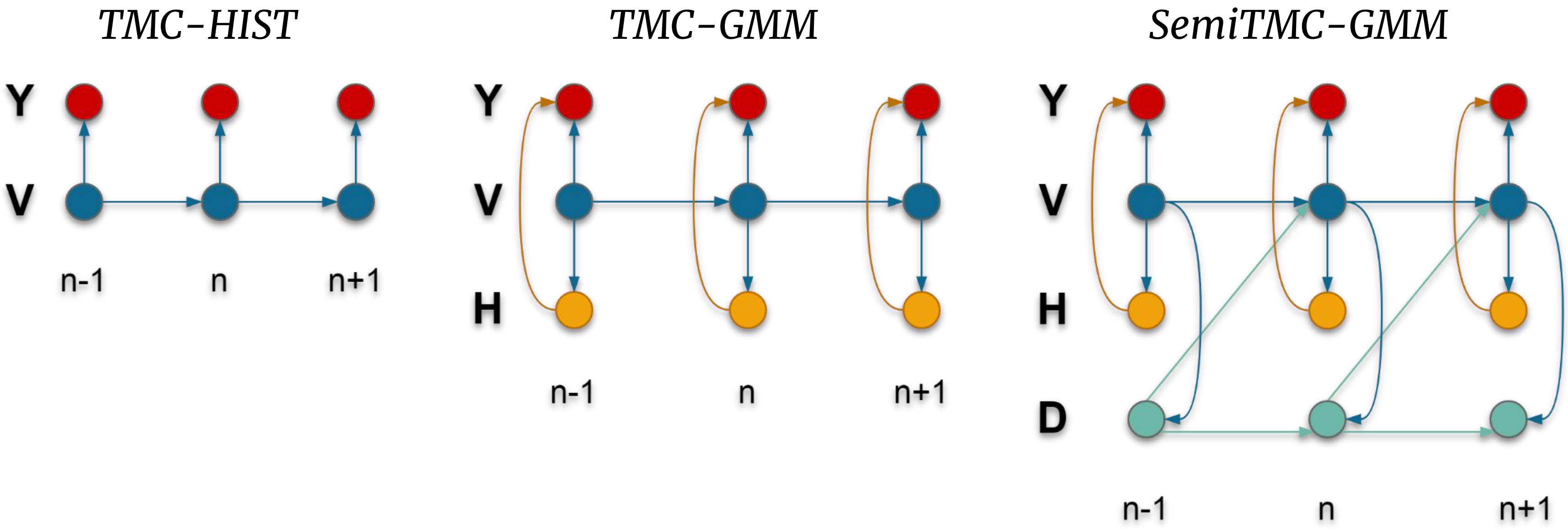
$$p(\boldsymbol{v}_{n+1} | \boldsymbol{z}_n, d_n) = \begin{cases} \delta_{\boldsymbol{v}_n}(\boldsymbol{v}_{n+1}), & d_n > 0 \\ p^*(\boldsymbol{v}_{n+1} | \boldsymbol{v}_n), & d_n = 0 \end{cases}$$

$*\delta_a(b) = 1 \quad when \quad a = b$

$\delta_a(b) = 0 \quad when \quad a \neq b$

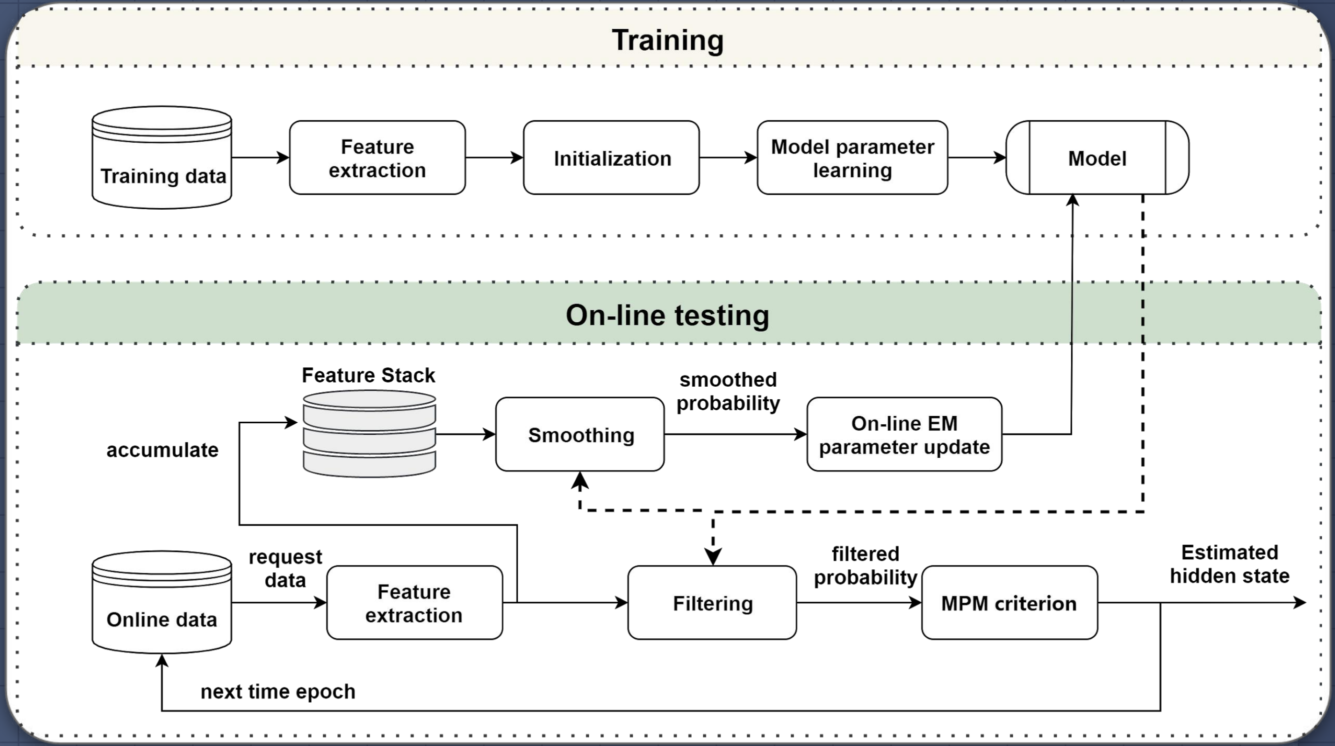
$$p(d_{n+1} | \boldsymbol{v}_{n+1}, d_n) = \begin{cases} \delta_{d_n - 1}(d_{n+1}), & d_n > 0 \\ p(d_{n+1} | \boldsymbol{v}_{n+1}), & d_n = 0 \end{cases}$$

State dependency graphs of TMC-HIST, TMC-GMM and SemiTMC-GMM.

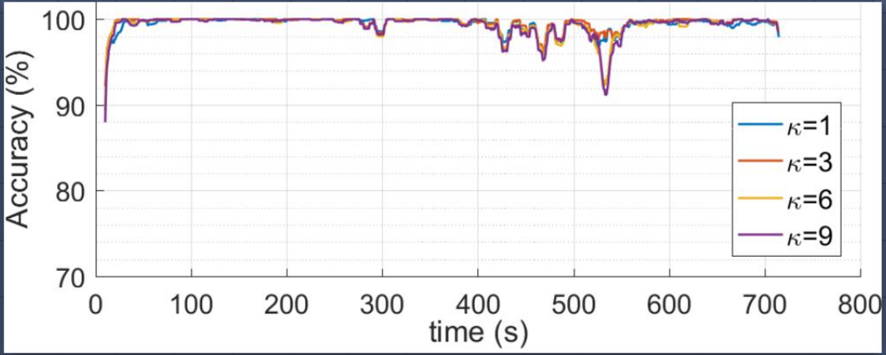


Compared to the on-line procedures of TMC-HIST, gait complete detection is no longer needed. Also, the estimated hidden states (activities and gait phases) are obtained from the filtered probability.

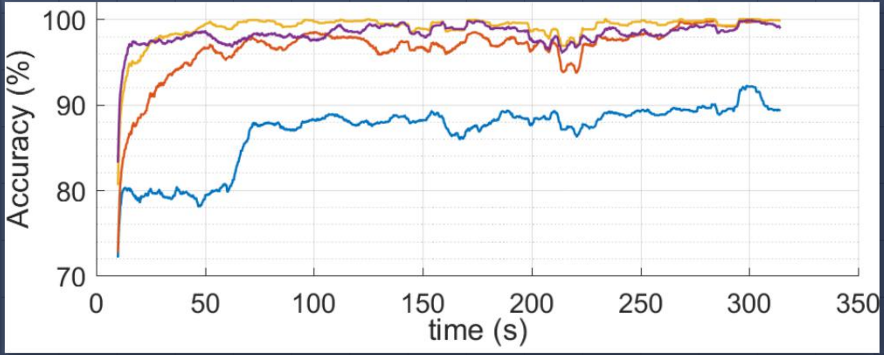
This means that SemiTMC-GMM is more robust than TMC-HIST.



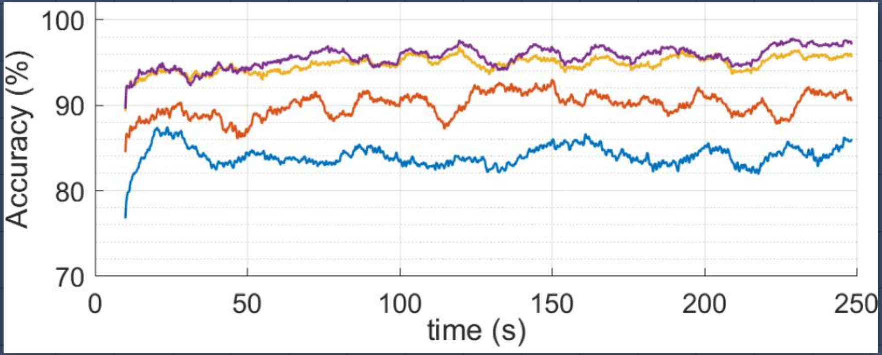
Walk



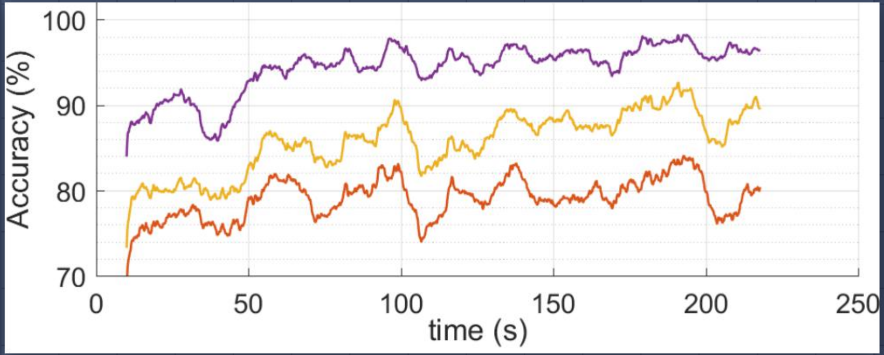
Run



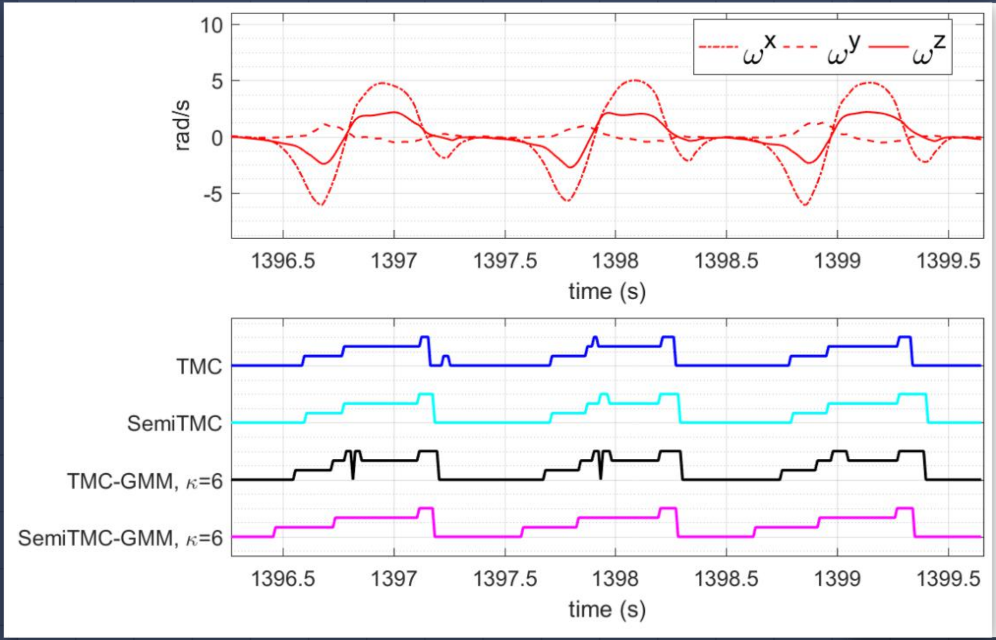
Stair ascent



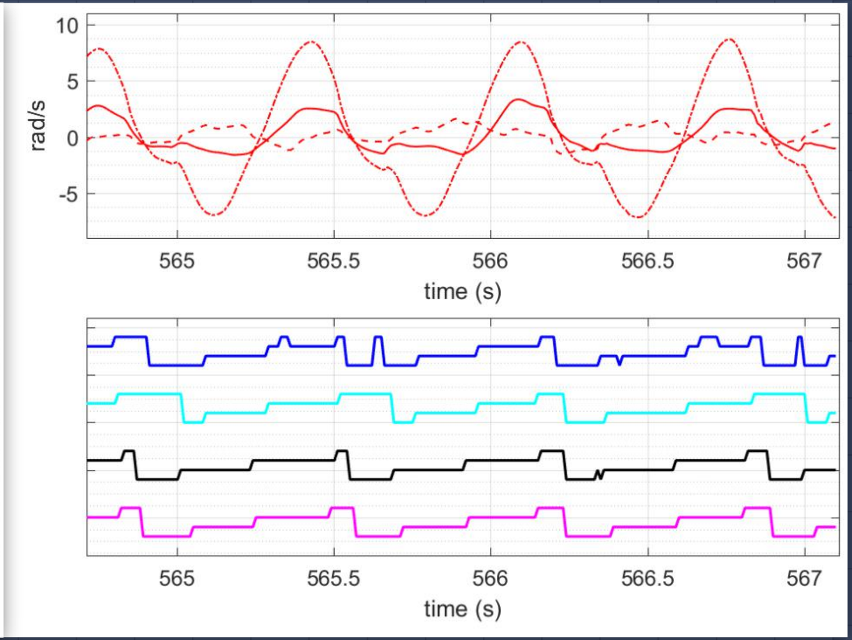
Stair descent



Detected gait phases of on-line mode of our dataset.



Walk



Run

Thanks, any question?

*Because of the non-parametric histograms in TMC-HIST, Baum-Welch algorithm is no longer suitable. "**Iterative Conditional Estimation**" (ICE) is used instead to learn the non-parametric model.*

Procedures of ICE:

1. *Model initialization.*
2. *Perform the forward and backward procedure to obtain* $\gamma_n(\boldsymbol{v}_n)$ *and* $\xi_n(\boldsymbol{v}_n, \boldsymbol{v}_{n+1})$
   Compute the transition probability:

$$p\left(\boldsymbol{v}_{n+1} \middle| \boldsymbol{v}_n, \boldsymbol{y}_1^N\right) = \frac{\xi_n(\boldsymbol{v}_n, \boldsymbol{v}_{n+1})}{\gamma_n(\boldsymbol{v}_n)}$$

3. *Simulate a realization of state sequence via* $\gamma_1(\boldsymbol{v}_1)$ *and* $p\left(\boldsymbol{v}_{n+1} \middle| \boldsymbol{v}_n, \boldsymbol{y}_1^N\right)$
4. *Update the parameters according to the simulated hidden states and observations.*
5. *Repeat Steps 2-3 until reach the maximum iteration.*

The Baum–Welch algorithm–based parameters learning for SemiTMC–GMM is in the following:

Estimation

$$\gamma_n(k) = p\left((\boldsymbol{v}_n, d_n) = k|\boldsymbol{y}_1^N\right) = \frac{\alpha_n(k)\beta_n(k)}{\sum_{k' \in \Lambda\times\Gamma\times L} \alpha_n(k')\beta_n(k')} \qquad \tilde{\gamma}_n(i) = \sum_{d_n} \gamma_n((i, d_n)) = \sum_{d_n} p\left(\boldsymbol{v}_n = i, d_n|\boldsymbol{y}_1^N\right)$$

$$\tilde{\gamma}_n(i,j) = \tilde{\gamma}(i) \cdot \frac{c_{ij} p(\boldsymbol{y}_n|\boldsymbol{v}_n = i, h_n = j)}{\sum_{j' \in K} c_{ij'} p(\boldsymbol{y}_n|\boldsymbol{v}_n = i, h_n = j')}$$

$$\xi_n(l,k) = \frac{\alpha_n(l) \cdot p(\boldsymbol{y}_{n+1}, h_{n+1}, (\boldsymbol{v}_{n+1}, d_{n+1}) = k|\boldsymbol{y}_n, h_n, (\boldsymbol{v}_n, d_n) = l) \cdot \beta_{n+1}(k)}{\sum_{l',k' \in \Lambda\times\Gamma\times L} \{\alpha_n(l') \cdot p(\boldsymbol{y}_{n+1}, h_{n+1}, (\boldsymbol{v}_{n+1}, d_{n+1}) = k'|\boldsymbol{y}_n, h_n, (\boldsymbol{v}_n, d_n) = l') \cdot \beta_{n+1}(k')\}}$$

Maximization

$$\zeta_k = \gamma_1(k) \qquad a_{lk} = \sum_{n=1}^{N-1} \xi_n(l,k) \Big/ \sum_{n=1}^{N-1} \gamma_n(l) \qquad c_{ij} = \sum_{n=1}^{N} \tilde{\gamma}_n(i,j) \Big/ \sum_{n=1}^{N} \tilde{\gamma}_n(i)$$

$$\boldsymbol{\mu}_{ij} = \sum_{n=1}^{N} \tilde{\gamma}_n(i,j)\boldsymbol{y}_n \Big/ \sum_{n=1}^{N} \tilde{\gamma}_n(i,j) \qquad \Sigma_{ij} = \sum_{n=1}^{N} \tilde{\gamma}_n(i,j)(\boldsymbol{y}_n - \boldsymbol{\mu}_{ij})^\top(\boldsymbol{y}_n - \boldsymbol{\mu}_{ij}) \Big/ \sum_{n=1}^{N} \tilde{\gamma}_n(i,j)$$

For parametric model, the parameters can be analytically calculated, using the on-line EM-based algorithm.

Statistic

$$s_{n'} = \left\{ s^{(1)}_{n',lk}, s^{(2)}_{n',k}, s^{(3)}_{n',ij}, s^{(4)}_{n',ij}, s^{(5)}_{n',ij} \right\}$$

$$s^{(1)}_{n',lk} = 1\left\{(\boldsymbol{v}_{n'}, d_{n'}) = l, (\boldsymbol{v}_{n'+1}, d_{n'+1}) = k\right\}$$

$$s^{(2)}_{n',k} = 1\left\{(\boldsymbol{v}_{n'}, d_{n'}) = k\right\}$$

$$s^{(3)}_{n',ij} = 1\left\{\boldsymbol{v}_{n'} = i, h_{n'} = j\right\}$$

$$s^{(4)}_{n',ij} = 1\left\{\boldsymbol{v}_{n'} = i, h_{n'} = j\right\} \boldsymbol{y}_{n'}$$

$$s^{(5)}_{n',ij} = 1\left\{\boldsymbol{v}_{n'} = i, h_{n'} = j\right\} \boldsymbol{y}^{\top}_{n'} \boldsymbol{y}_{n'}$$

Sufficient statistic

$$S_n = \frac{1}{n} \mathbf{E}_{\theta} \left(\sum_{n'=1}^{n} s_{n'} \right) | \boldsymbol{y}_1^n$$

Update

$$S_{n+1} = (1 - \rho_{n+1}) \cdot S_n + \rho_{n+1} \cdot \mathbf{E}_{\theta_n} \left(s_{n+1} | \boldsymbol{y}_{n+1} \right)$$

Parameters

$$\tilde{S}^{(2)}_{n,i} = \sum_{d_n} S^{(2)}_{n,(i,d_n)} \qquad c_{n,ij} = S^{(3)}_{n,ij} / \tilde{S}^{(2)}_{n,i}$$

$$\zeta_k = S^{(2)}_{1,k} \qquad \mu_{n,ij} = S^{(4)}_{n,ij} / S^{(3)}_{n,ij}$$

$$a_{n,lk} = S^{(1)}_{n,lk} / S^{(2)}_{n,k} \qquad \Sigma_{n,ij} = S^{(5)}_{n,ij} / S^{(3)}_{n,ij} - \mu^{\top}_{n,ij} \boldsymbol{\mu}_{n,ij}$$