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[N47] Variability of the Predicted Reverberation Times of a Concert Hall Induced by Absorption Coefficient Uncertainties

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ABSTRACT

This study is concerned with the acoustical properties of the concert hall Maurice Ravel in Lyon (France). As the reverberation time is still considered as the most important objective quantity in room acoustics, we focus on its evaluation by computer simulation. This is achieved by using the so-called radiosity method based on geometrical acoustics implemented in a software especially designed for this task, CeReS. The predicted reverberation time highly depends on the absorption coefficients values. Unfortunately, these coefficients are generally inaccurately known. For quantifying the induced uncertainty, we used an original statistical method. This method allows us (i) to predict the mean value and the standard deviation of the predicted reverberation time and (ii) to quantify the respective effect of each absorption coefficient. Also, maps of the SPL on the hall are discussed.

KEYWORDS: Room Acoustics, Simulation, Absorption, Uncertainty, Statistical

INTRODUCTION

The improvement of sound quality in concert hall is of great interest since Antiquity. Despite numerical methods have been developed for the modeling of room acoustics, this issue is often solved with empirical and experimental methods. Most of times, measurements are difficult to achieve and costly expensive for passable results. Design of concert hall has to be done with numerical methods. As the dimensions of concert hall are big, classical "low frequency" numerical methods like boundary elements method (BEM) or finite elements methods (FEM) are ineffective. "High frequency" methods such as ray tracing technique or statistical energy analysis (SEA) must be employed. Among them, the so-called radiosity method based on radiative transfer equation is another effective one. The method enables to determine the energy repartition inside the hall and others parameters such as reverberation times, early decay rate along audible frequency range. We used this method implemented in software named CeRes for this study.

Building an accurate modeling of such systems is not easy because it has to describe all sound mechanisms appearing in room acoustics, reflections, transmissions, diffusion and diffraction. Diffusion and diffraction are related to the boundaries geometry, the frequency of incident waves and the incidence angle. All these parameters are well known and easy to describe in modeling. Reflection and transmission are related to acoustical and mechanical properties of boundary materials. These properties are not obviously known because they are related to the real acoustic field and specific of the concert hall. Moreover, used materials become more and more complex (composite or sandwich). The real absorption could only be defined with measurements done in real conditions. As a consequence, absorption coefficients may be modeled as uncertain parameters.

The aim of this study is to quantify the variability of acoustic fields due to a lack of knowledge of absorption coefficients. The auditorium Maurice Ravel in Lyon (France) is chosen as the studied concert hall. This work couples two numerical approaches: the first one is an acoustic modeling based on the radiosity method, and the other one is a statistical modeling of the absorption coefficient uncertainty based on Taguchi's method. Some results are displayed and concern the reverberation time and sound pressure level map variabilities.

ACOUSTIC MODELING OF MAURICE RAVEL CONCERT HALL

Theoretical Description

The radiosity method was first developed by Kuttruff [1] and was deeply studied by Le Bot [2]. This method is based on the radiative transfer equation and is originally limited to the diffuse reflection (Lambert's law) although some extensions have been proposed for specularly reflecting surfaces. It intends to assess the reverberation time in rooms beyond the validity domain of Sabine's formula. Time reverberation calculation is deduced by a transient analysis. First, we describe the theoretical method for steady state analysis and we focus after on the transient analysis.

The steady state analysis is done in two steps: the first step computes the direct incident acoustic field and is easy to implement. The second step computes the reflected acoustic field. The total acoustic field is a linear summation of both direct and reflected fields (uncorrelated sources). The second step is then described. Let consider a room Ω of volume V enclosed by a surface Γ of area S and fulfilled with an acoustical fluid with a sound speed c and an attenuation factor m. The surface is assumed to be absorbing with a coefficient α defined as the ratio of reflected power over incident power and a reflection coefficient $\tau = 1-\alpha$. The reflection of energy is taken into account by some equivalent sources of magnitude $\sigma(q,t)$ distributed over surface Γ. At any point p at a distance R from q in direction θ, the radiative intensity is:

$$I(p,t) = \sigma(q, t - R/c)\cos\theta \frac{e^{-mR}}{4\pi R^2} \qquad (1)$$

Since the source is assumed to radiate energy following the cosine Lambert's law. R/c is the duration for the energy propagation from q to p. The incident power per unit surface when $p \in \Gamma$ is $I(p,t)\cos\varphi$, where φ is the incidence angle. In a second hand, the power emitted from p in all directions is given by integrating the intensity over a small sphere surrounding the source. It yields $\sigma(p,t)/4$. Now, at any point $p \in \Gamma$, the energy balance leads to $P_{refl} = \tau P_{inc}$ where P_{refl} is the reflected power (emitted from the equivalent source) and P_{inc} is the incident power from all other equivalent sources. Substituting the previous equations, the energy balance leads to:

$$\sigma(p,t) = \int_\Gamma \sigma(q, t - r/c)\tau(p)K(p,q)d\Gamma(q) \qquad (2)$$

Where

$$K(p,q) = \cos\theta\cos\varphi \frac{e^{-mR}}{\pi R^2} \qquad (3)$$

Until now, it has been assumed that the domain Ω is convex, that is to say all points $q \in \Gamma$ are viewed from the point p. Generally, some obstacles may lie inside Ω such that some points q may not contribute to the incident power at p. The function K in equation 3 must be multiplied by a visibility function V defined as V(p,q) = 1 if q is viewed from p and V(p,q) =0 otherwise. This formulation leads to an integral formulation on energy solved by a collocation scheme on surface S. A geometrical description of the surface with boundary elements is achieved for this purpose.

Transient analysis is derived from the steady state analysis. After the source is switched off, the energy decay follows an exponential law $\sigma(p,t) = \sigma(p)e^{-\lambda t}$. Substituting this equality in equation 2 leads to:

$$\sigma(p) = \int_\Gamma \sigma(q) e^{\lambda \frac{R}{c}} \tau(p) K(p,q) d\Gamma(q) \qquad (4)$$

Equation 4 is the radiative equation for reverberation time. The problem sums up as finding the constant λ which allows the existence of a non negative function σ. The software CeRes used a method proposed by Gilbert [3] to solve equation 4.

Modeling of Maurice Ravel Concert Hall

The Maurice Ravel concert hall has a volume of 22000 m^3. The overall area including walls, floor and ceilings is 6000 m^2. The experimental reverberation time over the audible frequency range is close to 3 seconds. The auditorium has been modeled by CeRes with the geometry displayed Figure 1. The mesh is done with 210 faces for a total of 1000 triangles. The enclosure is not convex, as a consequence the visibility function V must be carefully computed.

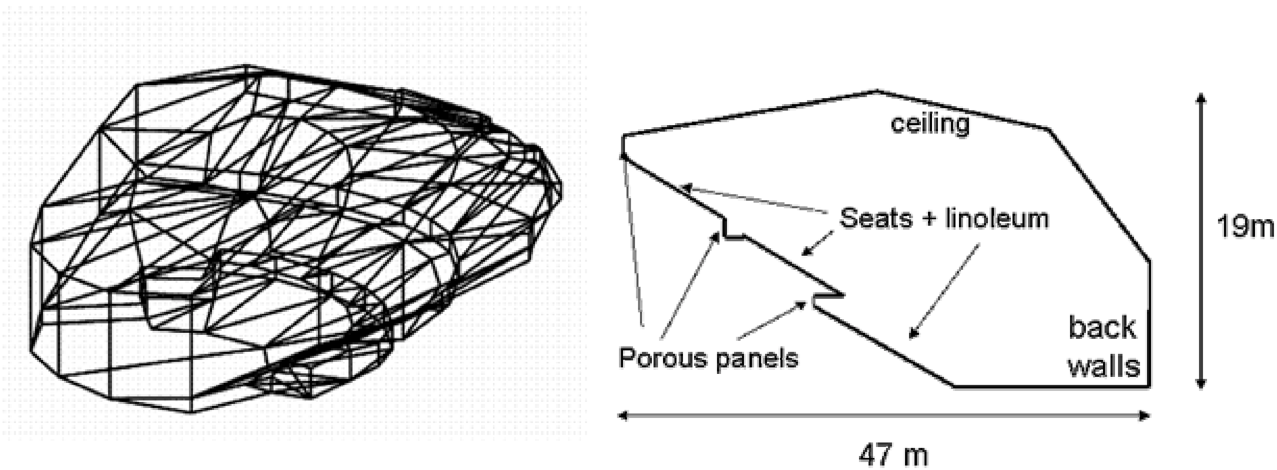

Figure 1: Mesh of the auditorium and absorption coefficients repartition.

Six different absorption coefficients have been introduced in the model: ceilings (staff), stage (hard wood), walls (porous panels), seats, floor (linoleum) and back walls (agglomerate plates). Their values aren't known accurately, especially for seats, walls and back walls. Indeed, in situ measurements have not yet been carried out. Their nominal values are given in Table 1 and are deduced from some tables available in the literature for an octave band around 1000 Hz.

Absorption coefficient	1000 Hz
Ceiling – α_1	0.1
Stage – α_2	0.05
Porous panels – α_3	0.51
Seats – α_4	0.24
Linoleum – α_5	0.03
Back walls – α_6	0.1

Table 1: Nominal values and corresponding absorption coefficients.

STATISTICAL MODELING OF ABSORPTION COEFFICIENTS UNCERTAINTY

Theoretical description of uncertainty

As some absorption coefficients aren't accurately known, we introduce uncertainty in the acoustic modeling of Maurice ravel concert hall. The uncertainty is modeled with random variables with known probability density function (PDF). Then the modified Taguchi's method [4, 5] is used to compute the statistical variations of reverberation time or sound pressure level repartition. A short description of modified Taguchi's method is given here.
Taguchi's method allows estimating in a very simple way the statistical moments of a function of multiple random variables whose PDF are known. Taguchi's method has been improved by D'Errico et al [4] for taking into account nonlinear effects as well. The modified Taguchi's method has been used for heat treatment problem [6]. Theoretical expressions for the first two moments of a function f({x}) of k randomly independent variables $\{x\}= (x_1,...,x_k)$ are:

$$E\left[f(\{x\})\right] = \int_{-\infty}^{+\infty} f(\{x\}) p_1(x_1)....p_k(x_k)dx_1...dx_k \tag{5}$$

$$\mathrm{var}\left[f(\{x\})\right] = \int_{-\infty}^{+\infty} (f(\{x\}) - E\left[f(\{x\})\right])^2 p_1(x_1)...p_k(x_k)dx_1...dx_k \tag{6}$$

In D'Errico's method, each PDF of random variable is sampled in three or more points and a weighting coefficient is assigned to each point depending of the kind of PDF. The response function is evaluated for all point combinations, which is equivalent to full factorial experiments with M responses or points combinations. This method is based on numerical integration of Gauss-Hermite quadrature for function of multiple variables. Mean value and variance of the function are estimated by linear combination of responses obtained previously with full factorial experiments as follows

$$E[f(\{x\})] = \sum_{i=1}^{M} W_i f_i \qquad (7)$$

$$\mathrm{var}[f(\{x\})] = \sum_{i=1}^{M} W_i (f_i - E[f(\{x\})])^2 \qquad (8)$$

where $W_i = \prod_{i=1}^{k} w_i$. For each uncertain variable, at least three samples are necessary to take into account nonlinear behavior of response function. Precision increases rapidly with the number of samples considered. In this study, $M=3^3=27$ is used to treat three random variables with 3 points. The principal advantages of this method are the ease of its numerical implementation and computational efficiency.

Statistical description of absorption coefficients

Only three coefficients are considered as uncertain α_3, α_4 and α_6 among the six ones. Indeed, α_1, α_2 and α_5 are built with well known materials. Each uncertain absorption coefficient is described by an uniform random variable whose interval bounds are given when compiling tables from the literature. The random variables are sampled with three points. Tables 2 gives the three values used for each coefficient. Absorption coefficient values for α_1, α_2 and α_5 are those given in Table 1. The acoustical computation is performed 27 times and the statistical results are computed at the end of the process.

	- α_3	- α_4	- α_6
Point 1	0.43	0.16	0.06
Point 2	0.51	0.24	0.1
Point 3	0.59	0.32	0.14

Table 2: Used values of uncertain absorption coefficients.

RESULTS

The results are presented when an omni directional source of 1 Watt is located on the stage. The frequency range is around 1000 Hz. The concert hall is empty, no public is present.

The first statistical moments of the reverberation times are as follows: mean value is equal to 3.03 seconds and standard deviation 0.38 seconds. The average value is in good agreement with experimental studies. The standard deviation is big because the input variability is not negligible. This result confirms the fact absorption coefficient values must be known accurately. A variance analysis of the 27 computations of reverberation time shows that seat's absorption coefficient is the most influent parameter. Unfortunately, this coefficient is the most difficult to obtain even with experimental results. Indeed this coefficient varies randomly and depends on the public repartition.
Let us now focus on the sound pressure level map. Figure 2 displays the points where acoustic pressure is computed.

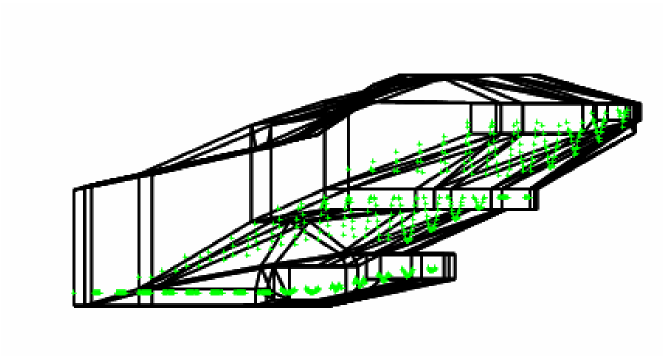

Figure 2: Location of points where SPL is computed..

Figure 3 displays the sound pressure level repartition into the concert hall for two absorbent configurations among the 27 computed. The acoustic field is not diffuse especially at the rear of the hall, and SPL are sensitive to absorption coefficient values. The SPL map for the less absorbent case seems to guarantee a better diffuse acoustic field, but the reverberation time is too long (3.8 seconds).

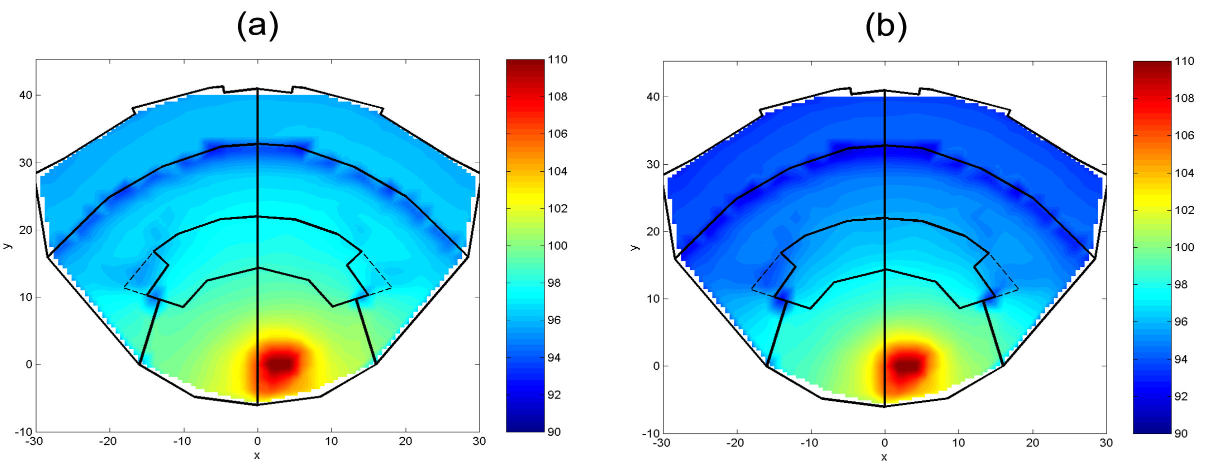

Figure 3: SPL maps for the less absorbent case (a) and for the more absorbent case (b).

CONCLUSION

This preliminary study proves that absorption coefficient values play a major part in concert hall design. The Taguchi's method is able to perform a sensitivity analysis and to predict the uncertainty induced by a lack of knowledge. This issue is solved by coupling two approaches: the first one is a high frequency modeling of the concert hall and the second is a statistical method modeling the uncertainty. More developments are in progress. This tool provides the sensitivity to a change of design.

REFERENCES

1. H. Kuttruff, *Room Acoustics* (Elsevier, 3rd edition, New York 1991)

2. A. Le Bot and A. Bocquillet, "Comparison of an Integral Equation on Energy and the Ray-Tracing Technique in Room Acoustics*", J. Acoust. Soc. Am.*, **108** (4), October, pp 1732-1740 (2000)

3. E. N. Gilbert, "An Iterative Calculation of Auditorium Reverberation", *J. Acoust. Soc. Am.*, **69** (1), pp 178-184 (1981)

4. J.R. D'Errico and N.A. Zaino jr., "Statistical Tolerancing using a Modification of Taguchi's Method", *Technometrics*, **30** (4), p. 397-405 (1988)

5. N. Driot, E. Rigaud, J. Sabot and J. Perret-Liaudet, "Allocation of Gear Tolerances to Minimize Gearbox Noise Variability", *Acta-Acustica*, **87**, pp 67-76 (2001)

6. J.C. Yu and K. Ishii, "A Robust Optimization Method for Systems with Significant Non Linear Effects", *Advances in Design Automation ASME*, **65** (1), pp.371-378 (1993)

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