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SEA: USING THE ACTIVE AND THE REACTIVE PART OF PURE TONE POWER INJECTED TO INVESTIGATE POWER FLOWS

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INTRODUCTION

Statistical Energy Analysis (SEA) is a method that can be used to investigate power flows in a dynamical system /1/. SEA requires one to evaluate the Damping Loss Factors (DLF's) and the Coupling Loss Factors (CLF's) of the system. Those modelling parameters can be identified by doing an experiment called "the power injection method" /2/. According to that method, one writes a balance on the power injected into the system. In agreement with SEA theory however, only the active part of mean power is considered in that balance. When the frequency band of analysis goes down to the lower frequency limit of the domain of SEA applicability, the power injection method often provides negative, non physical, parameters. Conservation of power injected into a system is a physical principle that is valid for the active and the reactive part of pure tone power. On the basis of that remark, we propose a new way for processing the output data of the power injection method. We apply that new way and the classical one to investigate power flows in a system made of two rods coupled in series. Both results are compared to those obtained analytically /3/.

USING THE ACTIVE AND THE REACTIVE PART OF PURE TONE POWER

Using only the active part. Let us consider a frequency band $[f_{min}\text{-}f_{max}]$ and an N part system. Let $P_j(f)$ be the active power injected at frequency f into part j and $P_{ij}^{diss}(f)$ be the power dissipated at frequency f in part i when part j alone is excited by $P_j(f)$. Depending on the kind of damping assumed, conservation of the active power injected into the system allows us to write :

$$\left\{P_j(f) = \sum_{i=1}^{N} P_{ij}^{diss}(f) \cong 4\pi f \sum_{i=1}^{N} \eta_i^h(f) E_{ij}^p(f)\right\} \text{ or } \left\{P_j(f) = \sum_{i=1}^{N} P_{ij}^{diss}(f) \cong 4\pi f \sum_{i=1}^{N} \eta_i^v(f) E_{ij}^c(f)\right\} \quad (1)$$

Where $\eta_i^h(f)$ is the pure tone hysteretical damping of part i. Despite the usual convention, we here adopt for symmetry a frequency dependent pure tone viscous damping. The one of part i is denoted by $\eta_i^v(f)$. $E_{ij}^p(f)$ and $E_{ij}^c(f)$ respectively are the pure tone potential and kinetic energies of part i when part j is excited. The two "$\cong$" signs mean that the damping of each part is neither totally hysteretical nor totally viscous is practical cases. By using a Fourier's analyser and today's transducers, we can easily measure the $P_j(f)$ and the $E_{ij}^c(f)$ at Nf discrete frequencies f_m in the band $[f_{min}\text{-}f_{max}]$. Since the $E_{ij}^p(f)$ are not as easily measurable nowadays, the use of them as input data for our method is not considered. For any frequency f_m, the here above two equations are valid and the second one of them can be rewritten as a linear system of N equations :

$$\left[E^{c}_{ij}(f_m)\right]_N^N\left[\eta^{v}_{i}(f_m)\right]_1^N = \left[\frac{P_j(f_m)}{4\pi f_m}\right]_1^N \tag{2}$$

By solving (2) for each frequency f_m, we get for each f_m the viscous damping of all the parts. On the contrary, since the $E^{p}_{ij}(f_m)$ are not easily measurable, we can not in practice determine the $\eta^{h}_{i}(f_m)$ by inverting the first one of equations (1).

Using the active and the reactive part. For any frequency f_m, the $E^{p}_{ij}(f_m)$ and the $\eta^{h}_{i}(f_m)$ are positive. For any f_m, one can therefore write by using the first one of equations (1) :

$$\min_{1\le i\le N} (\eta^{h}_{i}(f_m)) \sum_{i=1}^{N} E^{p}_{ij}(f_m) \le \frac{P_j(f_m)}{4\pi f_m} \le \max_{1\le i\le N} (\eta^{h}_{i}(f_m)) \sum_{i=1}^{N} E^{p}_{ij}(f_m) \tag{3}$$

Let $Q_j(f)$ be the pure tone reactive power injected into part j. Let $L_{ij}(f)$ be the pure tone lagrangian energy of part i when part j alone is excited. The principle of conservation of the reactive power injected into the system allows us to write for any frequency f_m /3/ :

$$Q_j(f_m) = 4\pi f_m \sum_{i=1}^{N} L_{ij}(f_m) = 4\pi f_m \sum_{i=1}^{N}\left[E^{c}_{ij}(f_m) - E^{p}_{ij}(f_m)\right] \tag{4}$$

Let us denote by α_{jm} the following expression :

$$\alpha_{jm} = \frac{P_j(f_m)}{4\pi f_m}\frac{1}{\sum_{i=1}^{N} E^{p}_{ij}(f_m)} = \frac{P_j(f_m)}{4\pi f_m}\frac{1}{\left[\sum_{i=1}^{N} E^{c}_{ij}(f_m) - \frac{Q_j(f_m)}{4\pi f_m}\right]} \tag{5}$$

Where $\sum_{i=1}^{N} E^{p}_{ij}(f_m)$ has been rewritten in terms of the $E^{c}_{ij}(f_m)$ and of the $Q_j(f_m)$ (see equation (4)). After dividing equation (3) by $\sum_{i=1}^{N} E^{p}_{ij}(f_m)$ and by introducing α_{jm}, one gets the following upper and lower bounds respectively for $\min_{1\le i\le N} (\eta^{h}_{i}(f_m))$ and for $\max_{1\le i\le N} (\eta^{h}_{i}(f_m))$:

$$\left\{\min_{1\le i\le N} (\eta^{h}_{i}(f_m)) \le \min_{1\le j\le N} (\alpha_{jm})\right\} \quad \text{and} \quad \left\{\max_{1\le j\le N} (\alpha_{jm}) \le \max_{1\le i\le N} (\eta^{h}_{i}(f_m))\right\} \tag{6}$$

After measuring the kinetic energies $E^{c}_{ij}(f_m)$, the active powers $P_j(f_m)$ and the reactive powers $Q_j(f_m)$ for each frequency f_m, we can for each f_m determine the α_{jm} and derive bounds for $\min_{1\le i\le N} (\eta^{h}_{i}(f_m))$ and for $\max_{1\le i\le N} (\eta^{h}_{i}(f_m))$. Those bounds even allow one to compute the $\eta^{h}_{i}(f_m)$ for each f_m, in the particular case where all the parts have the same hysteretical damping

APPLICATION TO THE CASE OF TWO RODS COUPLED IN SERIES

Let us now simulate the application of our new procedure and the one of the classical power injection method for investigating power flows in the case of the system presented in figure 1.

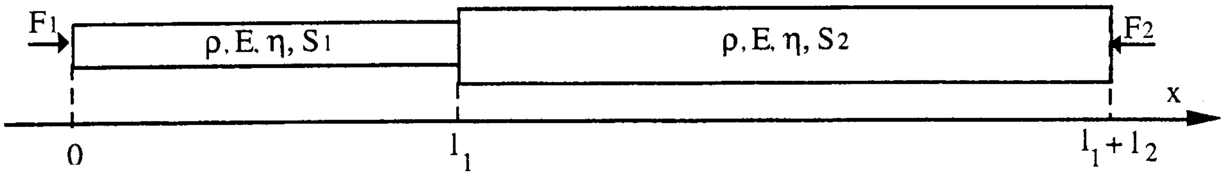

Figure 1 : system made of two rods coupled in series excited by two uncorrelated forces

That system is excited by two uncorrelated forces F_1 and F_2 in the octave band centred on f_o=2000 Hz. Each rod is considered as one part in the sense of SEA (N=2). For illustration of our method, we want to evaluate the mean active power $\overline{P_1^{diss}}$ dissipated in rod 1. $\overline{P_1^{diss}}$ will be displayed as a function of Δ, the geometrical average of the modal overlaps of the two rods :

$$\Delta = \sqrt{\prod_{i=1}^{2}\left[\frac{\eta_i l_i}{\pi}\sqrt{\frac{\rho_i}{E_{oi}}}\right]} \tag{7}$$

Where l_i is the length of rod i, η_i, ρ_i and E_{oi} respectively are the pure tone damping, the density and the real part of Young's complex modulus E_i of rod i. Both rods are assumed to be made of steel so that : $\eta_1=\eta_2=\eta=10^{-4}$ (in the whole octave band) and $\rho_1=\rho_2=\rho=7800$ kg/m^3. Hysteretical damping being assumed, one can write : $E_1=E_2=E=2.10^{11}(1+\eta I)$ (with $I^2=-1$).

The sections of rod 1 and of rod 2 are respectively equal to 1.e-4 m^2 and 3.e-4 m^2. To avoid that geometry may induce any peculiar coïncidence between modes of rod 1 and modes of rod 2, $\frac{l_2}{l_1}$ is kept equal to prime number π^2 whatever the value of Δ. For Nf frequencies regularly spaced in the octave band of analysis (Nf=128), the one dimensional Helmholtz equation subject to the ad hoc force and displacement continuity conditions is first solved. The required pure tone potential and kinetic energies $E_{ij}^p(f_m)$ and $E_{ij}^c(f_m)$, the pure tone active and reactive powers $P_j(f_m)$ and $Q_j(f_m)$ are then calculated. Uncorrelation between F_1 and F_2 is simulated as following. The value taken by any energetic quantity when the system is excited by the two uncorrelated forces is computed by adding up two terms : the value taken by the quantity when rod 1 alone is excited and the one taken by the quantity when rod 2 alone is excited. The $E_{ij}^p(f_m)$, $E_{ij}^c(f_m)$, $P_j(f_m)$ and $Q_j(f_m)$are frequency averaged in the octave band to respectively provide the required $\overline{E_{ij}^p}$, $\overline{E_{ij}^c}$, $\overline{P_j}$ and $\overline{Q_j}$. Equations (8), (9) and (10) are then used for computing $\overline{P_1^{diss}}$ analytically (8), by using the classical power injection method (9) and by using only the active part of pure tone power injected (10). Knowledge of the potential energies is assumed only in equation (8). In equation (9), η_1^{SEA} denotes the DLF of part 1, as determined by using the classical power injection method. The pure tone viscous damping in equation (10) is determined by applying equation (2). In our particular case where both rods have the same hysteretical damping, equation (6) exactly provides that damping $\eta^1(f_m)=\eta^2(f_m)=\eta(f_m)$ for each frequency f_m. Equation (11) differs from equation (10) only by the taking into account of that additional information.

$$\overline{P_1^{diss}} = \frac{4\pi}{Nf}\,\eta \sum_{m=1}^{Nf} f_m\left(E_{f1}^{p}(f_m) + E_{f2}^{p}(f_m)\right) \tag{8}$$

$$\overline{P_1^{diss}} = 4\pi f_0 \eta_1^{SEA}\left(\overline{E_{11}^{c}} + \overline{E_{12}^{c}}\right) \tag{9}$$

$$\overline{P_1^{diss}} = \frac{4\pi}{Nf}\sum_{m=1}^{Nf} \eta_1^{v}(f_m) f_m\left(E_{11}^{c}(f_m) + E_{12}^{c}(f_m)\right) \tag{10}$$

$$\overline{P_1^{diss}} = \frac{4\pi}{Nf}\sum_{m=1}^{Nf} \eta_1^{h}(f_m) f_m\left(E_{11}^{c}(f_m) + E_{12}^{c}(f_m)\right) \tag{11}$$

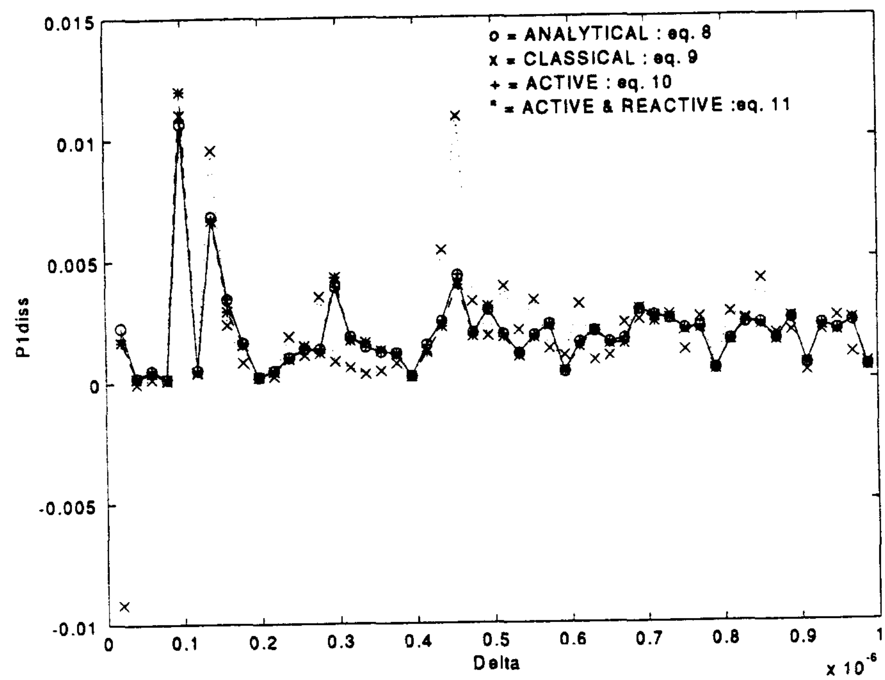

Figure 2 : $\overline{P_1^{diss}}$ *calculated by equations (8), (9), (10) and (11) as a function of* Δ

CONCLUSIONS

Power dissipated has been computed by four methods, as a function of one criteria for SEA applicability : the geometrical average of the modal overlaps of the rods, Δ. The classical power injection method leads to small random errors that may even become large for some specific values of Δ. It provides as expected negative dissipated power for low modal overlaps. Unlike the one obtained by using the classical method, both curves obtained by using the pure tone power injected fit the analytical curve. Although using both the active and the reactive part of power provides the exact hysteretical damping, this does not improve significantly the prediction.

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