GEOMETRICAL ACOUSTICS AND INTEGRAL REPRESENTATION OF ENERGY FIELDS

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1. INTRODUCTION

This paper is concerned with the study of a model well suited for high frequencies in acoustics or structural dynamics. In this frequency range, all travelling waves are assumed to be uncorrelated such that energy and intensity fields may be considered as additive quantities. The use of spherical waves and the application of a power balance leads to an integral equation, the solutions of which are compared with results of a software for room acoustics based on the ray-tracing technic.

2. REVIEW OF THE METHOD

This section briefly summarizes the main ideas of the integral method. All details which could be lacking may be found in References [1, 2, 3]. The variables of interest in view of a high frequency description are energy density W and energy flow vector $\mathbf{I}$. Since all waves are assumed to be uncorrelated, a superposition principle applies for energies. Thus, remarking that the acoustical pressure inside the domain Ω may be broken up into travelling waves stemming from both actual sources lying into the domain Ω and fictive sources located on the boundary $\partial\Omega$, the energy fields are:

$$W(M) = \int_\Omega \rho(S)G(S,M)dS + \int_{\partial\Omega} \sigma(P)G(P,M)cos\theta_{PM}dP \qquad (1)$$

$$\mathbf{I}(M) = \int_\Omega \rho(S)\mathbf{H}(S,M)dS + \int_{\partial\Omega} \sigma(P)\mathbf{H}(P,M)cos\theta_{PM}dP \qquad (2)$$

where $G(S,M) = e^{-mSM}/4\pi cSM^2$ and $\mathbf{H}(S,M) = e^{-mSM}\mathbf{SM}/4\pi SM^3$ are energy fields for spherical waves in three-dimensional space, θ_{PM} is the angle between the observation direction and the outward normal $\mathbf{n}_P$ to the boundary at P. c is the velocity of sound m an absorption factor. ρ is the power injected by actual sources and σ is the power of reflected by the boundary. Indeed ρ is assumed to be known but σ may be specified by an additional equation based on the power balance on the boundary. Introducing an absorption factor α depending on the nature of the constitutive material of the boundary, this equation is:

$$\sigma(P) = \frac{1-\alpha}{2c}[\int_{\Omega} \rho(S)\mathbf{H}(S,P)dS + \int_{\partial\Omega} \sigma(Q)\mathbf{H}(Q,P)cos\theta_{QP}dQ].\mathbf{n}_P \qquad (3)$$

This equation only states that the incident power at P is partially reflected into the ratio $1-\alpha$. Thus, the method may be drawn as follows, firstly the unknwon σ should be determined by solving the integral equation (3) either in a closed form or involving a numerical approach and, secondly, the energy fields are calculated in a direct way by applying the relationships (1,2).

3. APPLICATION TO ROOM ACOUSTICS

The application of the method previously described to room acoustics needs to solve a Fredholm integral equation of second kind for the boundary unknown σ. The construction of approximate solutions of such an equation is a well-studied problem. To choose the most adequate method, two facts may be considered. On the one hand, the kernel of the integral equation (3) is not singular since the incident power at a point P stemming from a point Q located in the vicinity of P is zero for a grazing incidence. This is an important fact which allows the use of an ordinary quadrature method for the evaluation of the occuring integrals. On the second hand, the unknown function may be discontinuous. This can be seen for instance by considering a non convex domain. A source does not illuminate the entire boundary and the point located in the boundary at the interface between the illuminated zone and the shadow zone is a discontinuous point for the incident power and then for the reflected power σ that is the unknown function.

In view of these arguments, the collocation method with constant unknowns on each subdomain has been choosen. The collocation points are located at the centre of each subdomain. The mesh of the boundary is obtained from a commercial software and can be of various shapes, triangles or quadrilaters for instance. Then the software that we have been developed for solving the integral equation may be applied to any room, convex or not, boundary of which can be divided into plane subdomains.

4. COMPARISON WITH THE RAY-TRACING TECHNIC

Some numerical simulations have been performed in order to compare the above method with the ray-tracing technic more widely used in room acoustics area. To this end, the sofware RAYON2.0 developed by EDF in France (department AMV-DER at Clamart), has been involved. The general characteristics are the following: 64000 rays are starting from the sound sources and may be reflected until 14 times on boundaries. Lambert's law is adopted for reflection in agreement with the similar assumption adopted in the integral method context. The direction of reflected rays is determined in a probabilistic way with respect to a Monte-Carlo simulation.

The first room in study has a L aspect and is 30X30X10 meters extended. It is drawn in a framework representation in the upper left part of Figure 1. The reflection coefficient is 0.75 uniformely on walls, floor and ceiling. The volume is 5000 m^3 and the modal density is 250 modes per rad/s at 1 kHz.

The second room is a factory of 20X10X10 meters. The reflection coefficients are 0.85, 0.75, 0.95 and 0.5 depending on the wall. The volume is smaller than the previous room so that the modal density falls to 125 modes per rad/s at 1 kHz.

5. CONCLUSION

This study points out the usefulness of the integral energetic model for application in room acoustics field. The main assumptions are the followings. Fields are constructed as a superposition of spherical waves which emerge from actual sources lying inside the domain and fictive sources lying in the boundary of the domain. All these waves are assumed to be uncorrelated which allows to add energies of each one.

The governing equations for energy field is a Fredholm integral equation of second kind which has been numerically solved with a standard collocation procedure with constant unknowns on each subdomain. A software has been developped to achieve this plan for any room for which the boundary is defined as plane subdomains.

Results of this software have been compared with those of a more classical method that is the ray-tracing technic. No phase is attached to rays and Lambert's law is adopted for reflection on walls. Physical assumptions of both models are quite similar and both methods may be considered as the same theory on a physical point of view.

The numerical simulations highlight this assertion since no significative difference can be observed. The agreement is almost perfect and the only few differences are due to numerical approximations.

Although numerical procedures of both methods are very different, the required CPU-time are of the same order with the subsequent nuance. For reverberent rooms, the ray-tracing algorithm converges slowly since a great amount of reflections should be accounted before the energy of a ray can be neglected. In opposition, when the absorption of walls is sufficient, the algorithm runs faster. In case of the integral method, the CPU-time does not depend on the absorption of walls. In all situations, the software must solve a linear system size of which is constant.

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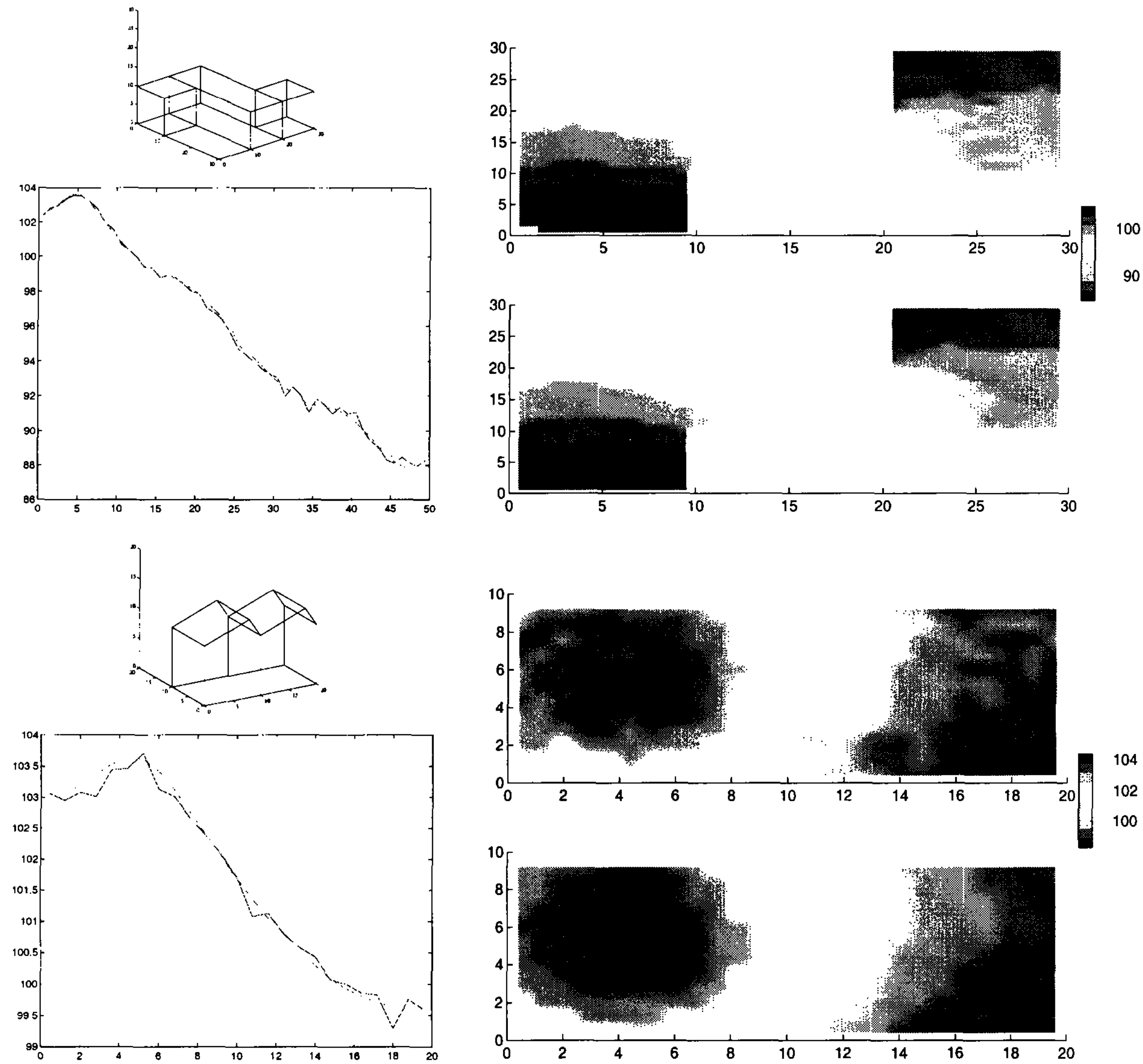

Figure 1: Comparison between results of integral method and the ray-tracing technic (code RAYON2.0 designed by EDF) for two rooms. Framework views of the rooms under study are drawn on left. In each case, two maps of the Sound Pressure Level (dB) inside the room are available on right. The upper one is from the ray-tracing technic and the lower one from the integral method. More direct comparisons of the Sound Pressure Level along a line crossing the room are on the left: dash line (smooth), integral method; solid line, ray-tracing.