Vibroacoustic prediction of mechanisms using a hybrid method

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Abstract

In this study, a hybrid approach is proposed to predict the noise radiated in the high frequency range by the casing of a mechanism vibrating in the low frequency range. This approach allows to couple a modal method for predicting the casing vibration, to a radiative transfer method well-suited for predicting the radiated noise. The radiative transfer method is a boundary integral method developed on the same assumptions as the known ray-tracing method. Equivalent energy sources are estimated on the casing surface from the surface pressure and velocity fields. The introduced technique is computing time-saving as the radiated noise is predicted on frequency bands. The validity of the approach is tested to predict the noise radiated by a vibrating ribbed aluminium plate. Comparisons are performed both with results obtained using the numerical boundary element method (BEM) and with experimental measurements.

1 Introduction

The prediction of noise radiated by a vibrating surface has given rise to the development of numerous methods depending on the context and above all depending on the frequency. The most famous and mainly used are the Finite and Boundary Element Method (FEM and BEM) [1,3] and the Statistical Energy Analysis (SEA) [2,14]. The FEM as the BEM are determinist methods well-suited for low frequency vibroacoustic problems but their applicability in the high frequency range is limited by the increasing size of the problem and the system uncertainties. Indeed, these methods relie on the well-known criteria of 6 elements within a wavelength such that the number of elements becomes drastic as the frequency increases and computing times become quickly prohibitive. The other problem deals with the geometry, material and damping uncertainties which influence the high frequency behaviour of the structure but that are not described in the determinist formulations of the FEM and the BEM. The SEA was developed to overcome the limitations inherent to the determinist methods and can solve some simple problems in the high frequency range.
Various approaches are proposed to combine methods valid in different frequency range, mainly in the field of vibrations. For instance, attempts are made to combine FEM and SEA analysis [4]. Vibroacoustic problems are classicaly solved by coupling the FEM for the simulation of the structural dynamics and the BEM for the prediction of acoustical field [1]. Recently, a sound radiation prediction approach was developed using the Power Flow Finite Element Method (PFFEM) [5] to predict the vibration energy density and intensity in the high frequency range and the Boundary Element Method (BEM) to predict the sound field radiated by the structure [13].
In this context, our purpose is to propose a hybrid approach to predict the noise radiated in the high frequency range by the casing of a mechanism which vibrates in the low frequency range. In a lot

of cases, mechanical parameters of mechanisms such as stiffeners properties are time dependent. For example, in the case of gears at stationnary running speed, meshing stiffnesses are characterized by periodic variations with high levels. Thus, specific modal method are used to predict the casing vibration. The radiative transfer method which is either an alternative to the Statistical Energy Analysis for vibration analysis [6,7] or to the ray-tracing technique for acoustical analysis [8] is here used to predict the sound field radiated by the casing of the mechanism. Equivalent energy sources are deduced from the knowledge of surface pressure and velocity fields. The approach to calculate acoustic sources is close to the one proposed in other works such as the Frequency Averaged Quadratic Pressure (FAQP) method [10], the Simplified Boundary Element Method which relies on the same concepts as the FAQP method [11], or the Energy Boundary Element Analysis (EBEA) [12]. Theoretical formulations of our hybrid method are exposed in this paper. The approach was first tested in the case of a free ribbed plate and is planned to be tested in presence of a mechanism and more specifically, in the case of a gearbox.

2 Formulations of the hybrid method

2.1 Energy formulation for the radiative transfer method

The radiative transfer method is known in the field of acoustics as the radiosity method and was first develop to predict the reverberation time of rooms beyond the validity of Sabine's formula [9]. This method relies on the assumptions that acoustical sources are uncorrelated. Thus, the acoustical energy density at any point $\mathbf{r}$ is the sum of two contributions, one coming from the known actual sources of power density ρ located inside the domain Ω and one coming from fictitious sources of power density σ located on the boundary S:

$$W(\mathbf{r}) = \int_\Omega \rho(\mathbf{p}, \mathbf{u_{pr}}) G(\mathbf{p}, \mathbf{r})\,\mathrm{d}\Omega + \int_S \sigma(\mathbf{q}, \mathbf{u_{qr}}) G(\mathbf{q}, \mathbf{r})\,\mathrm{d}S \tag{1}$$

$G(\mathbf{s}, \mathbf{r}) = e^{-mr}/4\pi\, c\, r^2$ is the energetic kernel for acoustical density which depends on the source-receiver distance $r = |\mathbf{s} - \mathbf{r}|$, on the sound speed c and on the absorption factor m. The amplitudes σ are unknown, and depend on the position of the source $\mathbf{q}$ and on $\mathbf{u_{qr}}$ which is the emission direction from $\mathbf{q}$ to $\mathbf{r}$. The boundary conditions lead to a Fredholm integral equation of second kind on the unknown σ.

2.2 Introduction of equivalent sources

Our work is aimed to predict the noise radiated by the casing of a vibrating mechanism and we consider here that the surface pressure and velocity fields are known. Acoustical energy sources are introduced on the surface of the casing and the purpose of our hybrid approach is to determine their amplitudes from the knowledge of the surface fields. To do that, let's consider the Helmholtz-Kirchhoff integral equation to calculate the pressure field radiated by a vibrating surface S:

$$p(\mathbf{r}) = \int_S p(\mathbf{q}) \frac{\partial g(\mathbf{q}, \mathbf{r}; \omega)}{\partial n} + \rho \gamma_n(\mathbf{q}) g(\mathbf{q}, \mathbf{r}; \omega)\,\mathrm{d}S \tag{2}$$

where ρ denotes the medium density and γ_n is the acceleration along the outward normal at point $\mathbf{q}$ on the vibrating surface. $g(\mathbf{q}, \mathbf{r}) = e^{-jkr}/4\pi r$ is the free-field Green's function, $r = |\mathbf{r} - \mathbf{q}|$ is the source-receiver distance and $k = \omega/c$ is the complex wavenumber. The time-averaged acoustic energy density W can be approximated by means of the frequency quadratic pressure in the following way:

$$W(\mathbf{r}) = \frac{|p(\mathbf{r})|^2}{2\rho c^2} \tag{3}$$

where c is the sound velocity. Thus, using the equation (2), the energy density is given by summing three contributions:

$$\begin{aligned} W(\mathbf{r}) &= \iint_S \frac{1}{2\rho c^2} p(\mathbf{q}) p^*(\mathbf{q}^{'}) \frac{\partial g(\mathbf{q},\mathbf{r};\omega)}{\partial n} \frac{\partial g^*(\mathbf{q}^{'},\mathbf{r};\omega)}{\partial n} \, \mathrm{d}S^{'} \, \mathrm{d}S \\ &+ \iint_S \frac{\rho}{2c^2} \gamma_n(\mathbf{q}) \gamma_n^*(\mathbf{q}^{'}) g(\mathbf{q},\mathbf{r};\omega) g^*(\mathbf{q}^{'},\mathbf{r};\omega) \, \mathrm{d}S^{'} \, \mathrm{d}S \\ &+ \Re\Big(\iint_S \frac{1}{c^2} \gamma_n(\mathbf{q}) p^*(\mathbf{q}^{'}) g(\mathbf{q},\mathbf{r};\omega) \frac{\partial g^*(\mathbf{q}^{'},\mathbf{r};\omega)}{\partial n} \, \mathrm{d}S^{'} \, \mathrm{d}S\Big) \end{aligned} \tag{4}$$

The hybrid approach is based on calculating the ensemble average (denoted $< . >$) of equation (4) using the fact that the Green's functions are determinist functions. Ensemble averaging is also used in the EBEA [13] assuming that only the cross-products taken at the same point are non-zero. The far-field approximation is used stating that $kr >> 1$, and the difference between r and $r^{'}$ is assumed to be small enough so that $r \simeq r^{'}$and thus $\frac{m}{2}(r + r^{'}) \simeq mr$. It follows that for instance the product $g(\mathbf{q},\mathbf{r};\omega) g^*(\mathbf{q}^{'},\mathbf{r};\omega)$ can be reduced to $e^{-jk(r-r^{'})} e^{-mr}/16\pi^2 r\, r^{'}$. Finally, under these conditions and noting that $< W >$ is a real quantity, three kinds of boundary sources with amplitudes σ_{pp}, $\sigma_{\gamma\gamma}$ and $\sigma_{p\gamma}$ are introduced: these sources are respectively called pressure sources, velocity sources and intensity sources. The acoustic energy due to their added contributions is written as:

$$< W(\mathbf{r}) > = \int_S (\sigma_{pp}(\mathbf{q},\mathbf{r}) + \sigma_{\gamma\gamma}(\mathbf{q},\mathbf{r}) + \sigma_{\gamma p}(\mathbf{q},\mathbf{r}))\, G(\mathbf{q},\mathbf{r}) \, \mathrm{d}S \tag{5}$$

where:

$$\sigma_{pp}(\mathbf{q},\mathbf{r}) = \Re\Big(\frac{k^2}{8\pi\rho c}\, r\, e^{-jkr}\, (\mathbf{u}_{qr} \cdot \mathbf{n}_q) \int_S R_{pp}(\mathbf{q},\mathbf{q}^{'}) \, \frac{e^{jkr^{'}}}{r^{'}} \, (\mathbf{u}_{q^{'}r} \cdot \mathbf{n}_{q^{'}}) \, \mathrm{d}S^{'}\Big) \tag{6}$$

$$\sigma_{\gamma\gamma}(\mathbf{q},\mathbf{r}) = \Re\Big(\frac{\rho}{8\pi c}\, r\, e^{-jkr} \int_S R_{\gamma\gamma}(\mathbf{q},\mathbf{q}^{'}) \, \frac{e^{jkr^{'}}}{r^{'}} \, \mathrm{d}S^{'}\Big) \tag{7}$$

$$\sigma_{\gamma p}(\mathbf{q},\mathbf{r}) = \Re\Big(\frac{-jk}{4\pi c}\, r\, e^{-jkr} \int_S R_{\gamma p}(\mathbf{q},\mathbf{q}^{'}) \, \frac{e^{jkr^{'}}}{r^{'}} \, (\mathbf{u}_{q^{'}r} \cdot \mathbf{n}_{q^{'}}) \, \mathrm{d}S^{'}\Big) \tag{8}$$

$\mathbf{n}_q$ is the unit outward normal to the surface of the casing at point $\mathbf{q}$. $R_{pp}(\mathbf{q},\mathbf{q}^{'}) =< p(\mathbf{q}) p^*(\mathbf{q}^{'}) >$, $R_{\gamma\gamma}(\mathbf{q},\mathbf{q}^{'}) =< \gamma_n(\mathbf{q}) \gamma_n^*(\mathbf{q}^{'}) >$, and $R_{\gamma p}(\mathbf{q},\mathbf{q}^{'}) =< \gamma_n(\mathbf{q}) p^*(\mathbf{q}^{'}) >$ are the pressure, acceleration and intensity cross-spectral densities that is the cross-correlation of two fields at points $\mathbf{q}$ and $\mathbf{q}^{'}$.
At this stage, source amplitudes do not depend only on the direction of emission but also on the position of the receiver point. This is not in agreement with the assumptions used in the radiative transfer method which are similar to those used in the ray-tracing technique, and lead to the equation (1) for the energy density. To overcome this problem, the difference of acoustical way $r - r^{'}$ is estimated in far-field as if plane waves were emitted at points $\mathbf{q}$ and $\mathbf{q}^{'}$. The algebric value of this approximated distance noted Δ is evaluated from geometrical considerations not detailed in this paper: it depends on $\mathbf{q}$ and $\mathbf{q}^{'}$ as well as on the direction of emission $\mathbf{u}_{qr}$ (Figure 1).
Under these conditions, the amplitudes σ_{pp}, $\sigma_{\gamma\gamma}$ and $\sigma_{p\gamma}$ can be written as:

$$\sigma_{pp}(\mathbf{q},\mathbf{u_{qr}}) = \Re\Big(\frac{k^2}{8\pi\rho c} \, (\mathbf{u}_{qr} \cdot \mathbf{n}_q) \int_S R_{pp}(\mathbf{q},\mathbf{q}^{'}) \, e^{jk\Delta} \, (\mathbf{u}_{q^{'}r} \cdot \mathbf{n}_{q^{'}}) \, \mathrm{d}S^{'}\Big) \tag{9}$$

$$\sigma_{\gamma\gamma}(\mathbf{q},\mathbf{u_{qr}}) = \Re\Big(\frac{\rho}{8\pi c} \int_S R_{\gamma\gamma}(\mathbf{q},\mathbf{q}^{'}) \, e^{jk\Delta} \, \mathrm{d}S^{'}\Big) \tag{10}$$

$$\sigma_{\gamma p}(\mathbf{q},\mathbf{u_{qr}}) = \Re\Big(\frac{-jk}{4\pi c} \int_S R_{\gamma p}(\mathbf{q},\mathbf{q}^{'}) \, e^{jk\Delta} \, (\mathbf{u}_{q^{'}r} \cdot \mathbf{n}_{q^{'}}) \, \mathrm{d}S^{'}\Big) \tag{11}$$

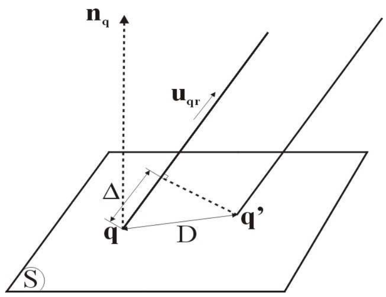


Figure 1: Notations - the difference of acoustical way between points $\mathbf{q}$ and $\mathbf{q}^{'}$ is estimated in the far-field as if plane waves were emitted.

Let's observe that the cross-term between surface fields are not neglected in these formulations. The SPL is evaluated using the common relation:

$$L_p = 20\log(\frac{p_{eff}}{p_o}) = L_I = 10\log(\frac{I}{I_o}) \tag{12}$$

with the adequated reference levels which are $p_o = 2.10^5$ Pa and $I_o = 10^{-12}$W.m^{-2}.

3 Prediction of noise radiated by a vibrating ribbed plate

3.1 Experimental setup

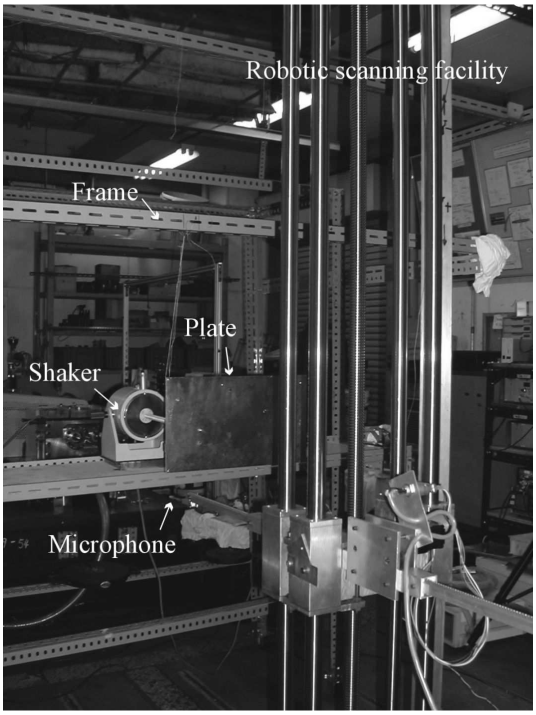
Surface fields measurements were performed on a vibrating ribbed plate to test the validity of the hybrid approach. The experiment did consist in scanning the sound pressure field over two parallel planes with a $1/4$" microphone receiver. This task is realized experimentally thanks to a robotic scanning facility which allows to move the microphone along the three coordinates axis. The ribbed plate is suspended to a frame specially built for the experiment approximating free boundary conditions. The plate is made with cast aluminium, its dimensions are 400 mm length, 230 mm width, 10 mm thick at the ribs and 5 mm thick everywhere else. A surface accelerometer located at x=20 mm, y=225 mm from the left bottom corner of the plate gives the reference signal for phase measurements. Data from the microphone and from the accelerometer are collected with a FFT analyzer connected to a computer which provides the data recording. The test rig is shown figure 2. The experiment did consist in driving a shaker located behind the plate with a broad-band signal so that the frequency range of 0-5 kHz was covered with a single pulse. Two measurement planes were chosen so that the particle velocity midway between them could be calculated. The distance between the plate and the first measurement plane is 10 mm and the distance between the two measurement planes is 10 mm. These distance are small enough to consider that surface fields are measured by this way. Both measurement planes are centered on the center of the plate.

3.2 Pressure and velocity measurements on the plate surface

The measurement of the pressure field on two parallel closely spaced planes allows to compute the gradient, and then the particule velocity. The sound pressure on the two measurement planes is given by:

$$p_1(t) = p_1 e^{j(\omega t+\phi_1)} \tag{13}$$
$$p_2(t) = p_2 e^{j(\omega t+\phi_2)} \tag{14}$$

(a)



(b)

Figure 2: Ribbed plate used for the experiment (a) and picture of the measuring system (b): the plate is suspended to the frame in front of a robotic scanning facility moving a microphone.

where p_1 and p_2 are the pressure amplitudes and ϕ_1 and ϕ_2 are the pressure phases respectively on the first and the second measurement plane. ω is the angular frequency. The pressure midway between the measurement planes is given by:

$$p(t) = \frac{(p_1(t) + p_2(t))}{2} \tag{15}$$

and the particle velocity midway between the measurement planes is given by:

$$u(t) = ue^{j(\omega t + \phi_u)} \tag{16}$$

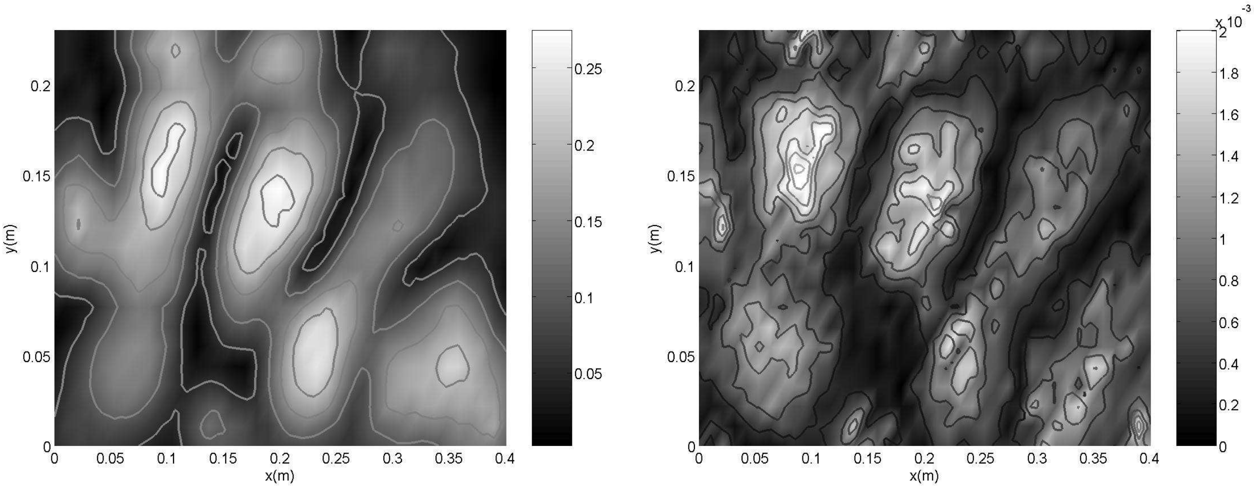
$$u = \frac{1}{\rho \Delta r \omega}\sqrt{p_1^2 + p_2^2 - 2p_1p_2cos(\phi_1 - \phi_2)} \tag{17}$$

$$\phi_u = arg((p_1 \sin\phi_1 - p_2 \sin\phi_2) + j(p_2 \cos\phi_2 - p_1 \cos\phi_1)) \tag{18}$$

where ρ is the density of air and Δr is the distance between the measurement planes. The maps presented on figure 3 show the pressure (a) and the velocity (b) fields measured over the plate at 2500 Hz.

3.3 Measurement and calculation of the SPL on a parallel plate

The SPL on a plane parallel to the ribbed plate is computed from the hybrid approach presented in this paper. The distance between the plane and the plate is 1 m (Figure 4). In order to check the validity of the results, a comparison with results from the Boundary Element Method implemented in the commercial software Sysnoise© is performed. In both cases, the pressure and velocity surface fields measured are used as entry data for the calculations. The ensemble averaging of the hybrid approach is assumed to be equivalent to the frequency averaging over the third-octave band. Thus, the analysis



(a) (b)

Figure 3: Pressure in Pa (a) and velocity in m/s (b) on the plate surface. The frequency is 2500 Hz.

is carried out on the third-octave band centered on 2500 Hz and BEM results that are obtained at pure tone frequencies are frequency-averaged within this band. Besides, experimental measurements realized on this same plane are presented as another reference. The figure 5 presents the maps of the SPL computed from the hybrid approach (a), from the BEM (b) and from experimental results (c) as well as the comparison of the results along a diagonal line of the measurement plane (d). Results obtained with the numerical methods are very similar and the global trend is in agreement with the experimental measurements.

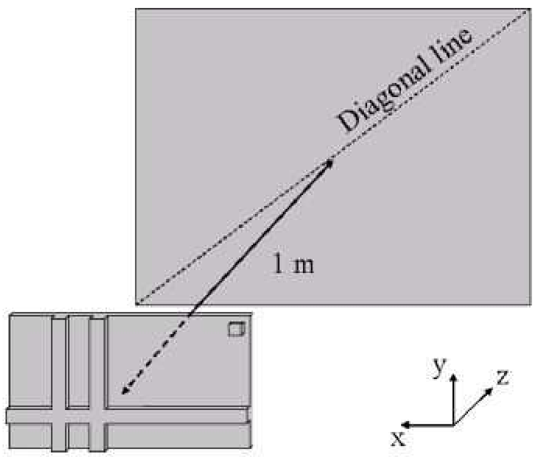
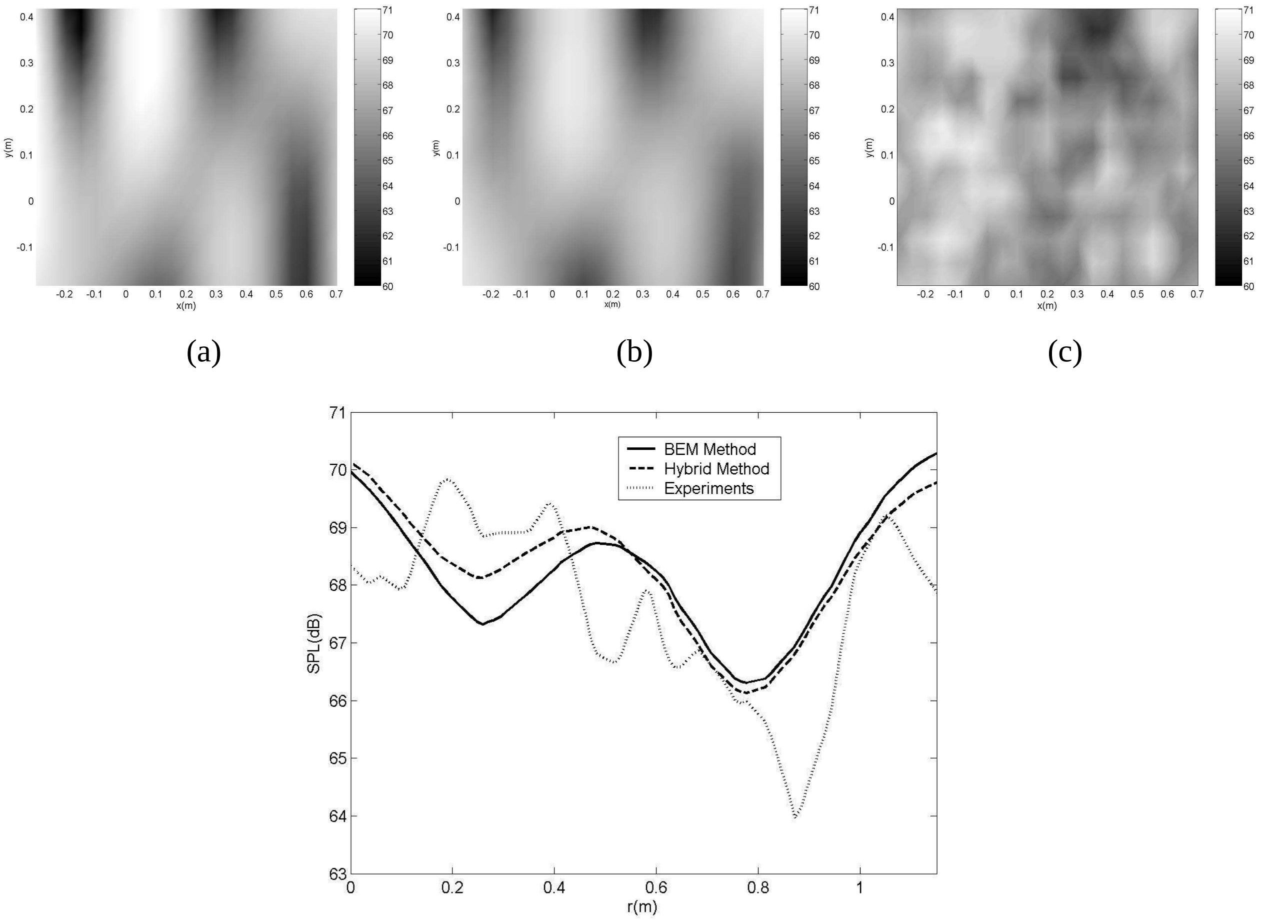


Figure 4: Measurement and calculation of the SPL on a parallel plane at 1 m from the ribbed plate: results are compared along the diagonal line of the measurement plane.

4 Conclusion

A vibroacoustic hybrid approach is proposed in this paper to solve the complete fluid-structure problem for acoustic radiation using a modal method for predicting the surface vibration and a radiative transfer method for predicting the radiated noise. In this approach, equivalent energy sources are introduced to implement the radiative transfer method and analytical formulations for their amplitude and directivity are proposed from the knowledge of the surface fields. Compared to the conventional



(a) (b) (c)

(d)

Figure 5: Maps of the SPL in dB (ref: 2.10^{-5} Pa) on the parallel plane computed using the BEM (a), the hybrid approach (b) and experimental measurements (c). Comparison between the results along the diagonal line of the measurement plane (d).

boundary element method, calculations are performed on third-octave bands and thus this method requires less computing times. Besides, the radiative transfer method is well-suited for high frequency applications such that this method is interesting to extend validity domain for acoustic field prediction. The approach was favourably tested in the case of a structure which is a ribbed plate, but theoretical developments remain valid for a mechanism as long as the vibratory behaviour of its casing may be determined. Therefore, ongoing work focuses on the noise radiated by a gearbox: the vibratory field on the casing is predicted using a specific modal approach and the hybrid method is applied to predict the noise radiated by the gearbox using the radiative transfer method.

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