Application of radiative transfer to vibroacoustics

A. Le Bot
Laboratoire de tribologie et dynamique des systèmes,
École centrale de Lyon,
36, avenue Guy de Collongue, 69134 ECULLY Cedex, FRANCE

Abstract

Few methods dedicate to the high frequency range in vibroacoustics. Apart from SEA which is the most popular method, some energy methods, among other approaches, attempt to fulfill this lack. This paper focuses on one of these methods, the application of radiative transfer equations to describe vibrational and acoustical fields in terms of rays. It is shown that this analogy leads to some integral equations well-suited to predict high frequency energy fields in both transient and steady states.

1 Introduction

In high frequency range, classical methods such as Finite Element Method or Boundary Element Method applied to governing equations are limited in practice due to the increasing number of elements. There is then an interest in developing some asymptotic methods which overcome this difficulty. SEA is certainly the most popular [1]. Based on energy concept, SEA is a very simple framework for analysing exchanges of energy between sub-structures.
Beyond this global approach, some methods attempt to generalize SEA and in particular to predict the repartition of energy inside sub-structures. Among them, let cite the vibrational conductivity approach [2, 3, 4, 5, 6] based on an analogy with heat conduction in thermics. These equations are further discussed in Ref. [7]. Another intensity approach well-suited to acoustics is found in Refs. [8, 9]. Considering that the intensity is derived from a scalar and vector potentials and that only the scalar potential part is relevant in high frequency range, Poisson's equation applies to the scalar potential. This method is different from the vibrational conductivity approach since the scalar potential is not the energy density.
The method we present in this paper is rather based on ray concept. However diffraction is accounted for in the sense of Geometrical Theory of Diffraction [10]. A boundary integral equation on intensity was found to be equivalent to the ray-tracing technique [11, 12]. Such integral equation was previously used to predict time reverberation in room acoustics [13, 14] but also early decay time [15]. This method was successively extended to assembled plates [16], to specular reflection [17, 18], to radiation [19], to transmission through walls [20] and to diffraction in acoustics [21]. In this paper, we present a summary of all these integral and functional equations on energy and it is shown that they model almost all relevant phenomena in vibroacoustics.

2 Energy fields of rays

An energy representation of vibrating fields in high frequency range requires two basic quantities: the energy density W and the intensity or energy flow vector $\mathbf{I}$ defined as being the power per unit surface normal to the ray.
Direct fields emanating from an impulse at time τ and point $\mathbf{s}$ and propagating in unbounded medium

are noted G for energy density and $\mathbf{H}$ for intensity. They are,

$$G(\mathbf{s},\tau;\mathbf{r},t) = G(\mathbf{s},\mathbf{r})\delta(t-\tau-s/c), \quad (1)$$
$$\mathbf{H}(\mathbf{s},\tau;\mathbf{r},t) = \mathbf{H}(\mathbf{s},\mathbf{r})\delta(t-\tau-s/c), \quad (2)$$

where $s = |\mathbf{s}-\mathbf{r}|$ is the source-receiver distance. $G(\mathbf{s};\mathbf{r})$ et $\mathbf{H}(\mathbf{s};\mathbf{r})$ are stationary fields given by,

$$G(\mathbf{s},\mathbf{r}) = \frac{e^{-ms}}{\gamma_0 c\ s^{n-1}}, \quad (3)$$
$$\mathbf{H}(\mathbf{s},\mathbf{r}) = cG(\mathbf{s},\mathbf{r})\mathbf{u}, \quad (4)$$

where γ_0 is the solid angle of space, c the sound speed or more generally the group speed, m is the absorption factor of the medium and $\mathbf{u}$ is the unit vector from $\mathbf{s}$ to $\mathbf{r}$. In some equations, H will denote the magnitude of vector $\mathbf{H}$.

Complete fields W and $\mathbf{I}$ result from a linear superposition of direct fields stemming from volume sources with power density ρ ($\mathrm{W/m^3}$) located inside domain Ω, from surface sources with power density σ ($\mathrm{W/m^2}$) located over boundary Γ and also from some line sources ($\mathrm{W/m}$) μ on Δ whose contribution is related to diffraction by wedges. It yields,

$$W(\mathbf{r},t) = \int_\Omega \rho(\mathbf{s},t')G(\mathbf{s},\mathbf{r})\mathrm{d}\Omega_\mathbf{s} + \int_\Gamma \sigma(\mathbf{p},\mathbf{u}',t')G(\mathbf{p},\mathbf{r})\mathrm{d}\Gamma_\mathbf{p} + \int_\Delta \mu(\mathbf{p},\mathbf{u}',t')G(\mathbf{p},\mathbf{r})\mathrm{d}\Delta_\mathbf{p}, \quad (5)$$
$$\mathbf{I}(\mathbf{r},t) = \int_\Omega \rho(\mathbf{s},t')\mathbf{H}(\mathbf{s},\mathbf{r})\mathrm{d}\Omega_\mathbf{s} + \int_\Gamma \sigma(\mathbf{p},\mathbf{u}',t')\mathbf{H}(\mathbf{p},\mathbf{r})\mathrm{d}\Gamma_\mathbf{p} + \int_\Delta \mu(\mathbf{p},\mathbf{u}',t')\mathbf{H}(\mathbf{p},\mathbf{r})\mathrm{d}\Delta_\mathbf{p}, \quad (6)$$

where $t' = t - |\mathbf{s}-\mathbf{r}|/c$ or $t' = t - |\mathbf{p}-\mathbf{r}|/c$ is the time accounting for the propagation from the source $\mathbf{s}$ or $\mathbf{p}$ to the receiver $\mathbf{r}$ and $\mathbf{u}'$ the unit vector from $\mathbf{p}$ to $\mathbf{r}$.

A local power balance applies for fields W and $\mathbf{H}$,

$$\mathbf{div}.\mathbf{I} + \frac{\partial W}{\partial t} + mcW = \rho \quad (7)$$

where mcW is the power density being dissipated and ρ the power density being injected by volume sources. This equation is valid inside the domain Ω.

In some cases, it can be considered that surface sources σ radiate energy following Lambert's law,

$$\sigma(\mathbf{p},\mathbf{u},t) = \sigma(\mathbf{p},t)\cos\theta, \quad (8)$$

where θ is the emission angle at point $\mathbf{p}$ in direction $\mathbf{u}$ measured with the normal $\mathbf{n}$.

3 Reflection

When reflection of rays occurs on the boundary Γ, the unknown σ distributed over Γ must be interpreted as being the reflected power per unit area. Introducing a reflection efficiency R defined as reflected power over incident power and taking into account the contribution of all sources on incident power, the equation on σ is derived.

In case of diffuse reflection, it yields [11, 12],

$$\frac{\gamma}{\gamma_0}\sigma(\mathbf{p},t) = \left[\int_\Omega R\rho(\mathbf{s},t')\mathbf{H}(\mathbf{s},\mathbf{p})\mathrm{d}\Omega_\mathbf{s} + \int_\Gamma R\sigma(\mathbf{q},t')\cos\theta'\mathbf{H}(\mathbf{q},\mathbf{p})\mathrm{d}\Gamma_\mathbf{q}\right].\mathbf{n}, \quad (9)$$

where $\gamma = \int \cos\theta \mathrm{d}u$, θ' is the emission angle at point $\mathbf{q}$ and $\mathbf{n}$ is the unit outward normal to the boundary at $\mathbf{p}$. This is a Fredholm's integral equation of second kind on σ.

In case of specular reflection, the detailed energy balance is applied. The equation on σ for any emission direction $\mathbf{u}$ is now [18],

$$\frac{\sigma(\mathbf{p},\mathbf{u},t)}{\cos\theta} = R\left[\int_{\mathbf{p}'}^{\mathbf{p}} \rho(\mathbf{s},t')e^{-ms}ds + \frac{\sigma(\mathbf{p}',\mathbf{u}',t')}{\cos\theta'}e^{-mr}\right], \tag{10}$$

where $\mathbf{u}'$ is the incident direction which specularly reflects in direction $\mathbf{u}$. $\mathbf{p}'$ is the point on the boundary located in direction $\mathbf{u}'$ from $\mathbf{p}$ and $r = |\mathbf{p}' - \mathbf{p}|$. θ' is the emission angle at point $\mathbf{p}'$ in direction $\mathbf{u}'$. The integral in the right-hand side is evaluated on the straight segment $\mathbf{p}' - \mathbf{p}$ and $s = |\mathbf{s} - \mathbf{p}|$. This is a functional equation on σ.

4 Transmission

The problem of a wave transmission between two media with different group speeds, for instance, the transmission of flexural waves at a joint of adjacent plates or the transparency of acoustical waves through walls, is a simple generalization of previous case of reflection. Consider two or more media referenced by a subscript i. $R_{ji}(\mathbf{u})$ is the transmission efficiency from direction $\mathbf{u}$ in medium j to medium i defined as the transmitted power over incident power. When $i = j$, this is simply the reflection efficiency on the boundary in medium i.

When emitted energy can be considered as being diffuse, the reflection sources follow Lambert's law and equations on unknowns σ_i are then [16],

$$\frac{\gamma}{\gamma_0}\sigma_i(\mathbf{p},t) = \sum_j \left[\int_{\Omega_j} R_{ji}(\mathbf{u}')\rho_j(\mathbf{s},t')\mathbf{H}_j(\mathbf{s},\mathbf{p})\mathrm{d}\Omega_{\mathbf{s}} + \int_{\Gamma_j} R_{ji}(\mathbf{u}')\sigma_j(\mathbf{q},t')\cos\theta'\mathbf{H}_j(\mathbf{q},\mathbf{p})\mathrm{d}\Gamma_{\mathbf{q}}\right].\mathbf{n}_j, \tag{11}$$

$\mathbf{u}'$ being alternatively the unit vector from $\mathbf{s}$ to $\mathbf{p}$ and from $\mathbf{q}$ to $\mathbf{p}$. This is a set of Fredholm's integral equations of second kind.

In case of specular reflection and transmission, vectors $\mathbf{u}_i$ are associated directions in medium i. They are linked by Snell-Descartes relationships. The angle between $\mathbf{u}_i$ and the normal is noted θ_i. The detailed power balance reads [18],

$$\frac{\sigma_i(\mathbf{p},\mathbf{u}_i,t)}{\cos\theta_i} = \sum_j \left(\frac{c_j'}{c_i'}\right)^{n-1} R_{ji}(\mathbf{u}_j') \left[\int_{\mathbf{p}_j'}^{\mathbf{p}} \rho_j(\mathbf{s},t')e^{-m_j s}\mathrm{d}s + \frac{\sigma_j(\mathbf{p}_j',\mathbf{u}_j',t')}{\cos\theta_j'}e^{-m_j r_j}\right], \tag{12}$$

where c_i' is the phase speed in medium i. $\mathbf{u}_j'$ is the incident direction which specularly reflects in direction $\mathbf{u}_j$ and $r_j = |\mathbf{p}_j' - \mathbf{p}|$. This is a set of functional equations on unknowns σ_i.

5 Diffraction

Consider now the case of diffraction sources μ. $D(\mathbf{v},\mathbf{u})$ is an energetic diffraction coefficient depending on two variables, the incident direction $\mathbf{v}$ and the emission direction $\mathbf{u}$, defined as the ratio of the emitted power $d\mathcal{P}_{\text{emit}}$ per unit solid angle du about $\mathbf{u}$ and per unit length $d\nu$ of the edge, and the incident intensity I_{inc} stemming from $\mathbf{v}$,

$$D(\mathbf{v},\mathbf{u}) = \frac{1}{I_{\text{inc}}} \times \frac{\mathrm{d}\mathcal{P}_{\text{emit}}}{\mathrm{d}\nu\mathrm{d}u}. \tag{13}$$

An explicit relationship for D may be derived for instance by solving the canonical problem of plane wave incident upon the edge. The equation on μ is then obtained by applying the detailed power balance in any direction $\mathbf{u}$ [21].

$$\frac{\mu(\mathbf{p},\mathbf{u},t)}{\gamma_0} = \int_\Omega D\rho(\mathbf{s},t')H(\mathbf{s},\mathbf{p})\mathrm{d}\Omega_{\mathbf{s}} + \int_\Gamma D\sigma(\mathbf{q},\mathbf{u}',t')H(\mathbf{q},\mathbf{p})\mathrm{d}\Gamma_{\mathbf{q}} + \int_\Delta D\mu(\mathbf{q},\mathbf{u}',t')H(\mathbf{q},\mathbf{p})\mathrm{d}\Delta_{\mathbf{q}}. \tag{14}$$

This is a functional equation on the unknown μ.

6 Radiation

For radiation, we introduce the subscript s for quantities related to structure and the subscript a for acoustics. Radiation of sound may be explained by several phenomena [22].
Radiation by surface modes only occurs beyond the coincidence frequency when structural waves are supersonic. In such a situation, when travelling structural waves continuously loss some energy which is converted into acoustical waves. Then, an energetic radiation factor is defined as,

$$\Sigma_s(\mathbf{v},\mathbf{u}) = \frac{1}{I_{\text{inc}}} \times \frac{\mathrm{d}\mathcal{P}_{\text{emit}}}{\mathrm{d}S\mathrm{d}u}, \tag{15}$$

the ratio of radiated power per unit area and unit solid angle over the incident intensity. In the mean time, the absorption factor m_s for structure is the sum of a term due to internal losses $\eta\omega/c$ where η is the damping loss factor and ω the circular frequency, and a term due to the acoustical radiation. Indeed, this last term and Σ_s must be related by power balance. It is found that,

$$m_s = \frac{\eta\omega}{c} + \int \Sigma_s(\mathbf{v},\mathbf{u})\mathrm{d}u. \tag{16}$$

Usually, the last integral does not depend on direction $\mathbf{v}$ for isotropic structure. Now, the acoustical surface sources σ_a distributed over the structure provide this energy lost by the structure. We get [19],

$$\frac{\sigma_a(\mathbf{p},\mathbf{u},t)}{4\pi} = \int_{\Omega_s} \Sigma_s(\mathbf{u}',\mathbf{u})\rho_s(\mathbf{s},\mathbf{u}',t')H(\mathbf{s},\mathbf{p})\mathrm{d}\Omega_{\mathbf{s}} + \int_{\Gamma_s} \Sigma_s(\mathbf{u}',\mathbf{u})\sigma_s(\mathbf{q},\mathbf{u}',t')H(\mathbf{q},\mathbf{p})\mathrm{d}\Gamma_{\mathbf{q}}, \tag{17}$$

where Ω_s denotes the structural domain and Γ_s its boundary. Remark that $\mathbf{u}'$ lies inside Ω_s whereas $\mathbf{u}$ is any direction inward fluid.
Radiation by edge modes occurs at any frequencies. This is particular case of diffraction. When a structural wave impinges on the edge of the structure, it is partially reflected into structure itself and partially diffracted into acoustics. Thus, it must be considered that the edge has a reflection efficiency R_s less than unity and an energetic diffraction coefficient D_{sa} similar to Eq. (13). These coefficients are related by power balance,

$$R_s = 1 - \frac{1}{\mathbf{v}.\mathbf{n}} \int D_{sa}(\mathbf{v},\mathbf{u})\mathrm{d}u, \tag{18}$$

where $\mathbf{v}.\mathbf{n}$ is the cosine of the incidence angle. This reflection coefficient R_s depends on the incident direction $\mathbf{v}$. Furthermore, the energy converted into acoustical waves is emanated by some acoustical sources distributed along the edge of the structure. Their power per unit length μ_a is determined by,

$$\frac{\mu_a(\mathbf{p},\mathbf{u},t)}{4\pi} = \int_{\Omega_s} D_{sa}(\mathbf{u}',\mathbf{u})\rho_s(\mathbf{s},t')H(\mathbf{s},\mathbf{p})\mathrm{d}\Omega_{\mathbf{s}} + \int_{\Gamma_s} D_{sa}(\mathbf{u}',\mathbf{u})\sigma_s(\mathbf{q},\mathbf{u}',t')H(\mathbf{q},\mathbf{p})\mathrm{d}\Gamma_{\mathbf{q}}. \tag{19}$$

The power density μ_a is then related to structural sources ρ_s and σ_s.

7 Absorption

Absorption is the reciprocal problem of diffraction. When an acoustical ray impinges on a structure, it is partially reflected into acoustics and partially converted into structural rays. As for radiation, absorption may occur by surface or edge modes.

Firstly, absorption by surface modes requires to introduce an energetic absorption coefficient Σ_a defined as in Eq. (15) but where now $\mathbf{u}$ lies in the structure and $\mathbf{v}$ is any incident direction from fluid. The reflection efficiency R_a for acoustical wave is then less than unity. The power balance implies,

$$R_a(\mathbf{v}) = 1 - \frac{1}{\mathbf{v}.\mathbf{n}} \int \Sigma_a(\mathbf{v}, \mathbf{u}) \mathrm{d}u, \tag{20}$$

where $\mathbf{v}.\mathbf{n}$ is the cosine of the incidence angle. This power being absorbed is recovered and re-emitted by some structural sources ρ_s located inside the structure,

$$\frac{\rho_s(\mathbf{p}, \mathbf{u}, t)}{2\pi} = \int_{\Omega_a} \Sigma_a(\mathbf{u}', \mathbf{u}) \rho_a(\mathbf{s}, \mathbf{u}', t') H(\mathbf{s}, \mathbf{p}) \mathrm{d}\Omega_{\mathbf{s}} + \int_{\Gamma_a} \Sigma_a(\mathbf{u}', \mathbf{u}) \sigma_a(\mathbf{q}, \mathbf{u}', t') H(\mathbf{q}, \mathbf{p}) \mathrm{d}\Gamma_{\mathbf{q}}, \tag{21}$$

where now Ω_a denotes the acoustical domain and Γ_a its boundary.
Secondly, absorption by edges is a particular case of diffraction. The energetic diffraction coefficient D_{as} is once again defined as in Eq. (13). All the energy impinging on the edge is either diffracted into structural wave or diffracted into acoustical wave. The acoustical diffraction sources have ever been found in Eq. (14) and the structural diffraction sources σ_s distributed along the edge are given by,

$$\frac{\sigma_s(\mathbf{p}, \mathbf{u}, t)}{2\pi} = \int_{\Omega_a} D_{as}(\mathbf{u}', \mathbf{u}) \rho_a(\mathbf{s}, t') H(\mathbf{s}, \mathbf{p}) \mathrm{d}\Omega_{\mathbf{s}} + \int_{\Gamma_a} D_{as}(\mathbf{u}', \mathbf{u}) \sigma_a(\mathbf{q}, \mathbf{u}', t') H(\mathbf{q}, \mathbf{p}) \mathrm{d}\Gamma_{\mathbf{q}}. \tag{22}$$

The power densities σ_s are then related to acoustical sources ρ_a and σ_a.

8 Conclusion

In this paper, a formalism for an integral representation of rays fields has been presented. All classical phenomena of vibroacoustics that is reflection, transmission, diffraction, radiation by surface or edge modes and absorption are accounted for. The advantage of using an integral representation instead of classical ray-tracing technique more usual in room acoustics or in geometrical theory of diffraction, is that it allows the use of boundary element method. Many examples and applications are available in the relevant references.

References

[1] R.H. Lyon, 'Statistical Energy Analysis of Dynamical Systems: Theory and Application', Cambridge, Massachusetts, MIT Press, (1975).

[2] V.D. Belov, S.A. Rybak and B.D. Tartakovskii, Sov. Phys. Acoust., 'Propagation of vibrational energy in absorbing structures', **23**, 115-119, (1977).

[3] D.J. Nefske and S.H. Sung, NCA Publication, 'Power Flow Finite Element Analysis of Dynamic Systems: Basic Theory and Application to Beams', **3**, (1987).

[4] J.C. Wohlever and R.J. Bernhard, J. Sound Vib., 'Mechanical Energy Flow Models of Rods and Beams', **153**, 1-19, (1992).

[5] R.S. Langley, J. Sound Vib., 'On the Vibrational Conductivity Approach to High Frequency Dynamics for Two-dimensional Structural Components', **182**, 637-657, (1995).

[6] M. Djimadoum and J.L. Guyader, 'Possibilities to Generalize the Heat Transfer Approach to Vibration of Plates Problems', Inter-Noise'95, Newport Beach CA, (1995).

[7] A. Carcaterra and A. Sestieri, J. Sound Vib., ’Energy density equations and power flow in structures’, **188**(2), 269-282, (1995).

[8] J.C. Pascal, 2nd International Congress on Acoustic Intensity, CETIM, Senlis, FRANCE, ’Structure and patterns of acoustic intensity fields’, (1985).

[9] M. Thivant, ’Modélisation de la propagation acoustique par la méthode du potentiel d’intensité’, thèse 03ISAL0042, INSA Lyon, (2003).

[10] J.B. Keller, J. Opt. Soc. Am., ’Geometrical theory of diffraction’, **52**(2), 116-130, (1962).

[11] A. Le Bot and L. Ricol, ’Integral equation instead of heat conduction equation for medium and high frequencies’, Inter-Noise’95, Newport Beach USA, 579-582, (1995).

[12] A. Le Bot and A. Bocquillet, J. Acoust. Soc. Am., ’Comparison of an integral equation on energy and the ray-tracing technique for room acoustics’, **108**(4), 1732-1740, (2000).

[13] Kuttruff H., Simulierte Nachhallkurven in Rechteckräumen mit diffusem Schallfeld”, Acoustica”, 333-342”, **25**, (1971).

[14] Kuttruff H., J. Acoust. Soc. Am., ’A simple iteration scheme for the computation of decay constants in enclosures with diffusely reflecting boundaries’, **98**(1), 288-293, (1995).

[15] Miles R.N., J. Sound Vib., ’Sound field in a rectangular enclosure with diffusely reflecting boundaries’, **92**(2), 203-226, (1984).

[16] A. Le Bot, J. Sound Vib., ’Energy transfer for high frequencies in built-up structures’, **250**(2), 247-275, (2002).

[17] Kuttruff H., Acoustica with Acta Acoustica, ’Stationary propagation of sound energy in flat enclosures with partially diffuse surface reflection’, **86**(6), 1028-1033, (2000).

[18] A. Le Bot, J. Acoust. Soc. Am., ’A functional equation for the specular reflection of rays’, **112**(4), 1276-1287, (2002).

[19] V. Cotoni and A. Le Bot, International Journal of Acoustics and Vibration, ’Radiation of plane structures at high frequency using an energy method’, **6**(4), 209-214, (2001).

[20] V. Cotoni and A. Le Bot and L. Jezequel, Acustica with Acta Acustica, ’Sound transmission through a plate by an energy flow approach’, **88**(6), 827-836, (2002).

[21] E. Reboul and A. Le Bot and J. Perret-Liaudet, Comptes Rendus Mécanique, ’Introduction of acoustical diffraction in the radiative transfer method ’, **332**(7), 505-511, (2004).

[22] D.G. Crighton, J. Sound Vib., ’The 1988 Rayleigh medal lecture: fluid loading - the interaction between sound and vibration’, **133**(1), 1-27, (1989).