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[N454] Application of the radiosity method to acoustical diffraction in high frequency range

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ABSTRACT

Energy methods are of a great interest to solve acoustic radiation problems at high frequency range. One of these methods called ‘radiosity method’ has been applied in room acoustics’ to the calculation of reverberation times : it is based on the equation of the radiative energy transfer and deals with local energy quantities. But, only geometrical optical rays are taken into account in the ‘radiosity method’, and consequently diffraction phenomena are neglected. However, these phenomena lead to a redistribution of the incident energy that should be taken into account to well describe the repartition of sound pressure.

In this context, this paper aims to include diffraction phenomena into the radiosity method. A software called CeReS specially appropriate for this method was improved for the task and was used for calculations. An example of acoustical diffraction around corners is considered to assess the method, and results are compared to the Geometrical Theory of Diffraction (GTD) and to results given by the Boundary Element Method using the software Sysnoise ©.

KEYWORDS: Diffraction – Radiosity – High frequency

INTRODUCTION

The problem of acoustical diffraction has given rise to a lot of papers in the literature, whether it is question of diffraction behind plane screens, around wide barriers [1], or around corners [2]. The diffraction phenomena can be treated thanks to the Geometrical Theory of Diffraction developed by Keller in the 1950s [3]. This theory relies on the high frequency assumption that waves propagate like rays, so it appears as an extension of geometrical acoustics introducing diffracted rays.

This paper is aimed to study diffraction phenomena by means of another method called the radiosity method. This method has been investigated in room acoustics [4,5] and is based on radiative transfer equations. A recent study [6] was lead to adapt the radiosity method to perfectly specular reflection. This is going to be completed here with the introduction of diffraction effects. All cases studied here are two-dimensional.

Section 1 is dedicated to the theoretical formalism: power exchanges are investigated and energy balance leads to an expression of the energetic diffraction coefficient. The sound energy calculation is detailed in section 2. In sections 3 and 4, some numerical examples are presented to support the theory: a two-dimensional study of diffraction around corners is considered in section 3, a case of multiple diffraction is exhibited in section 4. Results are compared respectively with the Geometrical Theory of Diffraction (GTD), and with results given by the standard Boundary Element Method using the software Sysnoise ©.

CALCULATION OF THE ENERGETIC DIFFRACTION COEFFICIENT

Consider an incoming sound wave in the direction $\mathbf{v}$ impinging on a structure with a diffraction point O. The sound wave is then diffracted in the direction $\mathbf{u}$. The energetic diffraction coefficient $D(\mathbf{v},\mathbf{u})$ can thus be defined as follows:

$$P_{diff}(\mathbf{u}) = D(\mathbf{v},\mathbf{u}) I_{inc}(\mathbf{v}) \quad (1)$$

where $P_{diff}(\mathbf{u})$ is the specific intensity that is to say the diffracted power flow per unit solid angle in the direction $\mathbf{u}$ and $I_{inc}(\mathbf{v})$ is the incident power flow in the direction of propagation $\mathbf{v}$.

For an incident plane wave with an amplitude p_0 at the diffraction point, the diffracted wave is given by the GTD [2] and can be written:

$$p_{diff}(\mathbf{v},\mathbf{u})=p_0 d(\mathbf{v},\mathbf{u})\frac{e^{ikr}}{\sqrt{r}} \tag{2}$$

$$d(\mathbf{v},\mathbf{u})=\frac{e^{\frac{i\pi}{4}}}{\sqrt{2\pi k}}\nu\sin(\nu\pi)\left(\frac{1}{(\cos\nu\pi-\cos\nu(\varphi+\varphi_0))}+\frac{1}{(\cos\nu\pi-\cos\nu(\varphi-\varphi_0))}\right) \tag{3}$$

where $d(\mathbf{v},\mathbf{u})$ is the diffraction coefficient introduced by the GTD due to the source in the direction $\mathbf{v}$ and the observation point at position $(\mathbf{u},r)$. k denotes the acoustical wavenumber. $d(\mathbf{v},\mathbf{u})$ depends on the inner angle of the diffracting wedge 2Ω, ν is a wedge index defined as $\nu=\pi/(2\pi\text{-}2\Omega)$, φ and φ_0 give source and observation point positions defined such that the region exterior to the wedge is between $\varphi=0$ and $\varphi=2\pi\text{-}2\Omega$.

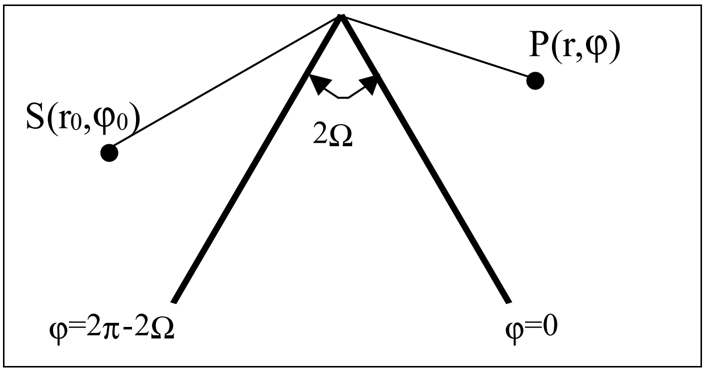


Figure 1: Source and observation point positions

The specific intensity for the diffracted wave can thus be written in terms of the impinging acoustical power flow I_{inc} in the direction $\mathbf{v}$:

$$P_{diff}(\mathbf{u})=|d(\mathbf{v},\mathbf{u})|^2 I_{inc}(\mathbf{v}) \tag{4}$$

Thus, the energetic diffraction coefficient can be written in terms of the diffraction coefficient introduced by the GTD:

$$D(\mathbf{v},\mathbf{u})=\left|d(\mathbf{v},\mathbf{u})\right|^2 \qquad (5)$$

SOUND ENERGY CALCULATION

The radiosity method relies on the assumption that all sources are uncorrelated so that the superposition principle can be applied to energy fields. Reflection and diffraction phenomena are taken into account so three kinds of power sources are introduced. The contribution of these sources to the whole acoustical field can be written thanks to the kernel function at point M defined in a 2D configuration as $G(r)=e^{-mr}/2\pi rc$ where r denotes the source-receiver distance, c is the sound speed, and m is the attenuation factor. From this point, the whole power flow at any point M $\mathbf{I}(M)$ is calculated by summing the power flow attached to the direct field $\mathbf{I}_{dir}(M)$, the power flow attached to the reflected field $\mathbf{I}_{refl}(M)$ and the power flow attached to the diffracted field $\mathbf{I}_{diff}(M)$:

$$\mathbf{I}(M)=\mathbf{I}_{dir}(M)+\mathbf{I}_{refl}(M)+\mathbf{I}_{diff}(M) \qquad (6)$$

$$\text{With: } \mathbf{I}_{diff}(M)=\sum_i c\sigma_i(\mathbf{u_i})G(r_i)\mathbf{u_i} \qquad (7)$$

where $\mathbf{u_i}$ and r_i are respectively the direction and the distance between the diffracting point P_i and the receiver point M.

Diffraction sources σ_i are determined thanks to the expression of energetic coefficient:

$$\sigma_i(\mathbf{u_i})=2\pi D(\mathbf{v},\mathbf{u_i})I_{inc}(P_i) \qquad (8)$$

The incident field I_{inc} takes into account all fields coming at the diffracting point that is to say the direct field, the reflected field and the diffracted field coming from the other diffracting points.

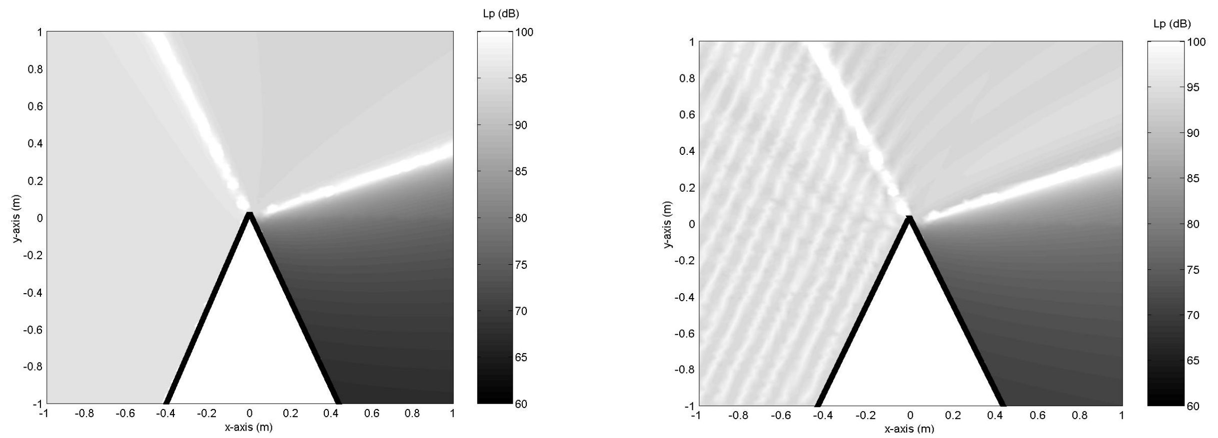
For the practical point of view, equation (6) is solved numerically and is applied here to two situations : the diffraction of plane wave at wedge, and the diffraction of cylindrical wave at edge of a half-plane.

NUMERICAL EXAMPLES

Diffraction at wedges

The first example considered here is the diffraction of a plane wave with unit amplitude around a rigid wedge. The wedge inner angle is $2\Omega = \pi/4$. The energetic reflection coefficient $R(\mathbf{v},\mathbf{u})$ is taken equal to 1 and the energetic diffraction coefficient $D(\mathbf{v},\mathbf{u})$ is given in equation (5).

Results for the energy density are presented in figure 2 and are compared to results obtained from analytical formulations of the GTD [2]. Figure 3 shows the repartition of the energy density in a linear scale along a representative line taken at y=0.2m. A good agreement between both methods is noticeable: interferences are not visible applying the radiosity method as they are not taken into account in the assumption of wave uncorrelation. At geometrical boundaries the diffraction coefficient tends to infinity: indeed, in these regions the acoustical field cannot be divided into geometrical and diffraction parts.



(a) (b)

Figure 2: Sound pressure level (Lp) in dB with the radiosity method (a) and with the GTD. Frequency : 2500 Hz.

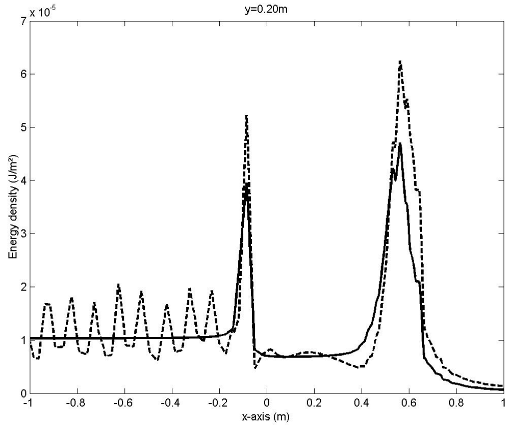


Figure 3: Energy density in linear scale with the radiosity method (-) and with the GTD(- -). Frequency : 2500 Hz.

Diffraction at edges of an half-plane

The second example is intended to illustrate a case of multiple diffraction. Consider a unit cylindrical point source emitting in front of a half-plane. Acoustical rays are reflected in the left half-plane and diffracted at the two edges of the half-plane facing the source. Diffraction sources σ are thus determined solving a linear system linking contributions of direct and diffraction sources. Results are presented in figure 4 and are compared to results obtained with the boundary element method using the software Sysnoise ©.

As in the previous case, variations of acoustical energy are quite well described with the radiosity method, interference effects are neglected. Geometrical boundaries are particularly visible because in these regions the diffraction coefficient tends to infinity, and so the sound pressure level is infinite. In these regions, the acoustical field cannot be described in terms of rays so the 'radiosity method' which is a ray method is not adapted. Such a problem does not occur with the Boundary Element Method.

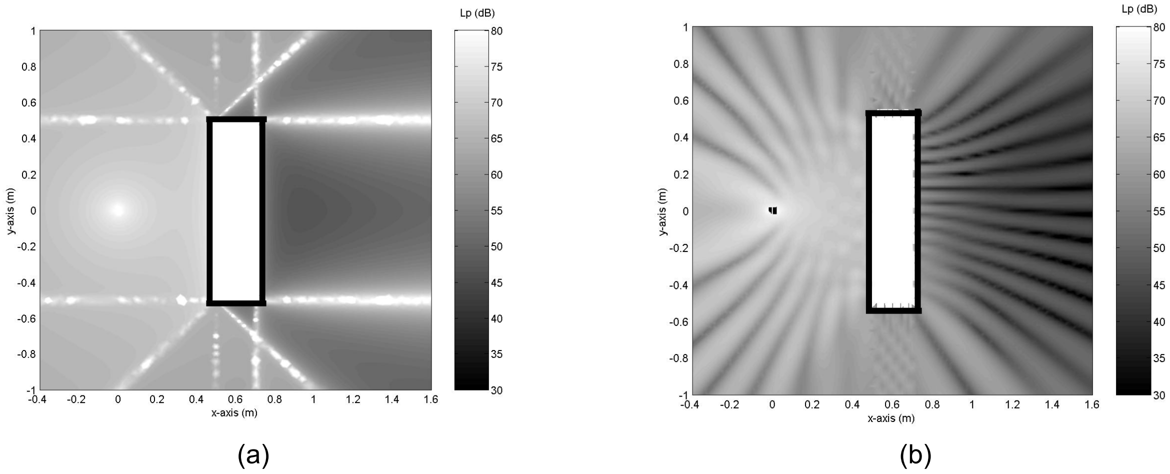


Figure 4: Sound pressure level (Lp) in dB with the radiosity method (a) and with the Boundary Element Method (Sysnoise ©). Frequency : 2500 Hz.

CONCLUSIONS

An expression for the energetic diffraction coefficient is proposed in this paper. This expression is interesting as it enables to introduce diffraction phenomena in the radiosity method. The results obtained with this method are comparable to those obtained with analytical formulations of the GTD or with the boundary element method : interference effects are not visible as the method relies on the assumption that wave are not correlated so that energy fields can be added. The limits of the method appear at geometrical boundaries where results obtained are not valid : indeed, in these regions the acoustical field cannot be described in terms of rays so the radiosity method as the GTD cannot be applied.
Thus, the work has now to be developped to well describe the acoustical field at these geometrical boundaries.

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