inter-noise 2002

The 2002 International Congress and Exposition on Noise Control Engineering
Dearborn, MI, USA. August 19-21, 2002

A Hybrid Model for the Prediction of Noise Radiated by Gearbox.

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Abstract

In this study, a hybrid model is proposed for the set gear system with its internal source, the housing and the surrounding cavity. A finite element model is built including gear with its periodic mesh stiffness, shafts, bearings and housing. An original method allowing the solving of parametrically excited systems is used to compute the vibratory field of the housing. The acoustical prediction of radiation is based on geometrical acoustics. However, the involved method leads to an integral equation which is solved by conventional methods rather than a ray-tracing technique. This is done by the software CeReS especially designed for this task. Finally, some conditions for the coupling are introduced.

1. Introduction

Despite many studies, the prediction of noise radiated by gearbox is still a complex problem. Firstly, the interaction process between the internal dynamical source (mainly the static transmission error) and the vibratory behaviour of the housing leads to some difficulties. It requires the use of some specific methods for the resolution. Secondly, the excitation spectrum is mainly beyond one kilohertz which is in the low frequency range in regard to the mass and stiffness of the gearbox. In contrast these frequencies are rather in the high frequency range in regard to the size of the surrounding acoustical cavity. This opposition requires to involve simultaneously some different methods.

The outline of the present paper is as follows. Section 2. is intended to describe the methods involved for the whole calculation of noise radiated by gearbox. In Section 3., the coupling conditions which relate the velocity field on the housing to the powers of acoustical sources are derived. Finally, a numerical example is presented in Section 4.

2. Review of the method

The main vibration source of gearboxes is generated by the meshing process as a consequence of the static transmission error (STE). STE is the difference between actual position of the output toothed wheel and the position it would occupy if the gear drive was perfect and infinitly rigid. It is mainly governed by periodic components at the meshing frequency due to (1) elastic deflections of gear teeth under load (periodic mesh stiffness) and (2) teeth geometry modifications, manufacturing errors and shaft misalignments. Under operating conditions, STE generates dynamic mesh forces transmitted to the housing through bearings.

Evaluation of STE is firstly based on a 3D finite element analysis of tooth deflections. Compliance matrix associated with nodes of tooth flanks are computed for each toothed wheel. Then, a non linear equation describing the static equilibrium of the gear pair is solved for a set of successive positions of the driving wheel [1].

An integrated model of the gearbox is built in order to compute its dynamic response. Helical gear, shafts, and housing are discretized using finite element method. Toothed wheels are coupled by a 12x12 stiffness matrix defined from the geometrical characteristics of the gear pair and from the mean value of mesh stiffness. Each tapered-roller element bearing is modelled by a 10x10 stiffness matrix. The matrix equation which governs forced vibrations of the discretized gearbox can be written as follows:

$$M\ddot{x}(t) + C\dot{x}(t) + Kx(t) + k(t)Dx(t) = E(t) \tag{1}$$

M and K are the classical mass and stiffness matrix provided by the finite element method; Matrix D is derived from geometric characteristics of the gear pair; $k(t)$ is the periodic mesh stiffness and $E(t)$ is an equivalent force vector induced by STE.

A modal analysis of the gearbox can be done from the time-invariant homogeneous counterpart equation. Damping is introduced using an equivalent viscous damping rate for each mode. Forced responses of gear, shafts and housing are computed simultaneously using a spectral and iterative method [2]. This one provides a direct spectral description of the vibratory response at each degree of freedom.

The method involved for the calculation of the acoustical radiation is now described. It is based on the geometrical acoustics. However, instead of using the ray-tracing technique, an integral formulation is prefered. All details of this method may be found in Refs. [3, 4, 5, 6]. The variables of interest are the acoustical energy density W and the energy flow vector $\mathbf{I}$. Since all acoustical sources are located on the structure and are assumed to be uncorrelated, the energy and intensity inside the medium are simply,

$$W(\mathbf{r}) = \int_S \sigma(\mathbf{p}) G(\mathbf{p}, \mathbf{r}) \cos\theta \, \mathrm{d}S_{\mathbf{p}} \tag{2}$$

$$\mathbf{I}(\mathbf{r}) = \int_S \sigma(\mathbf{p})\mathbf{H}(\mathbf{p},\mathbf{r})\cos\theta \mathrm{d}S_{\mathbf{p}} \tag{3}$$

where $G(\mathbf{p},\mathbf{r}) = e^{-mr}/4\pi c r^2$ and $\mathbf{H}(\mathbf{p},\mathbf{r}) = e^{-mr}\mathbf{u}/4\pi r^2$, $r = |\mathbf{r}-\mathbf{p}|$, $\mathbf{u}$ is the unit vector from $\mathbf{p}$ to $\mathbf{r}$, θ the emission angle, m an absorption factor. σ is the density of radiated power. The boundary condition leads to a Fredholm integral equation of seconf kind on unknowns σ. A specific software named CeReS has been designed to numerically solved this equation and to compute the Sound Pressure Level at any receiver point.

3. Coupling condition at the radiating surface

We start from a velocity field assumed to be known over the radiating surface. The problem is to locate the acoustical sources that radiate energy in far-field and also to evaluate their powers. All these sources should be uncorrelated.

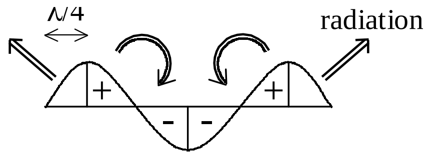

Figure 1: Radiation of sound is cancelled within the plate below the critical frequency.

However, it is well-known that in presence of sinusoidal deflection, positive parts cancel with neighboor negative parts (Figure 1). This phenomenon leads to the edge or corner radiation mode. From the case of a rectangular plate, the following properties may be summarized,

- acoustical cancellation arises between adjacent areas,
- the length of participating areas is $\lambda/4$,
- the velocity in these areas is in opposition of phase.

In general, it is a very difficult problem to find all acoustical cancellation for an arbitrary velocity field. This task can only be achieved by solving the governing equations for vibroacoustics without simplifying assumptions.

This effect can be captured by classical Boundary Element Method. But it cannot be calculated with an energy method such as the ray-tracing technique or all other methods which do not take into account the relative phase of the velocity field. Thus, the Boundary Element Method is used to compute the parietal pressure and, finally the normal intensity over the surface. In general, the map of normal intensity shows some separated radiating areas. An acoustical equivalent source is attached to each radiating area. The power of these sources is obtained by integrating the normal intensity over each area.

$$\sigma(\mathbf{p}) = \frac{P}{4}\delta(\mathbf{p}-\mathbf{s}), \quad P = \int_S I(\mathbf{r})\mathrm{d}S \tag{4}$$

where $\mathbf{s}$ is the position of the equivalent source.

4. Numerical simulation

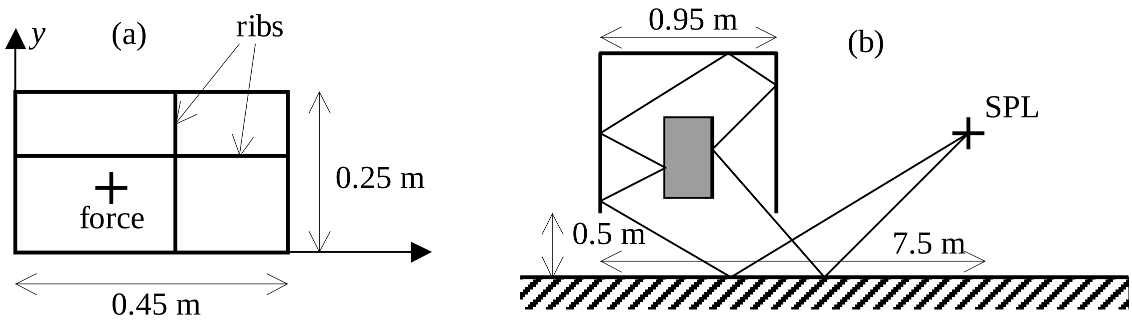

Figure 2: (a): View of the stiffened plate and the two ribs. (b): View of the gearbox with the radiating side (thick line), the surrounding cavity with absorbing walls, the road and the receiving points.

In order to show the feasibility of the proposed method, we consider a box with five infinitly rigid faces and a vibrating face as a model of the gearbox housing. The vibrating face is modelled with a stiffened plate. The plate has a 6 mm thickness, a 0.4 m length and a 0.25 m larger. The critical frequency of the equivalent infinite plate is 1933 Hz. Two ribs of 6 mm thickness and 2 cm height stiffen the plate as shown in Figure 2. The velocity field at 3000 Hz has been calculated with the Finite Element Method. The maps of the magnitude and phase of the velocity field are shown in Figure 4.

From the velocity field, a Boundary Element Method has been used to evaluate the normal active intensity over the plate. This calculation has been done under the assumption that the plate is baffled. The map of the resulting normal intensity is shown in Figure 5. Five equivalent acoustical sources are clearly obtained. Equivalent sources are taken to be the centered on the radiating area. The power is obtained by integrating the normal intensity over the radiating area. Power and position of these sources are summarized in the following table.

source	1	2	3	4	5
x (cm)	4	14	22	37	37
y (cm)	6	6	9	4	12
power (μW)	0.3	0.3	0.06	0.02	0.06

Table 1: Position and power of acoustical sources in the frame of Figure 2.

The acoustical problem is shown in Figure 2. The gearbox is a box of size 0.25 m, 0.45 m, 0.15 m with one radiating face (the right one). This box is inside an open cavity of size 0.65 m, 1.25 m, 0.95 m. The set gearbox-cavity 0.5 m above the road. The five acoustical sources have been distributed over the right face of the gearbox. The gearbox is assumed to reflect almost perfectly the acoustical rays ($\alpha = 0.01$) with a diffuse reflection law. An acoustical absorption factor $\alpha = 0.1$ is assumed for the interior of the cavity whereas a factor $\alpha = 0.04$ is adopted for the causeway. Now the Sound Pressure Level is evaluated at some receiver points along three lines located 1 m, 3 m and 7.5 m far from the vehicule. Results are shown in Figure 5b.

Conclusions

This study is an attempt to calculate the Sound Pressure Level far from a radiating gearbox. The model takes into account the internal excitation (Static Error Transmission) of the housing, the acoustical cavity where the gearbox radiates and the causeway where acoustical rays are reflected. Three methods have been involved, Finite Element Method for parametrically excited system for the velocity over the housing, Boundary Element Method for the normal intensity and geometrical acoustics for the exterior radiation problem. The full steps are summarizd in the diagram of Figure 3. The aim of this calculation is just to demonstrate the feasibility of such an approach, by the way the obtained values are not realistic for many reasons. But the authors now intend to improve the adopted approach and to compare the results with some reference calculations.

Acknowledgements

The authors acknowledge the Centre National de la Recherche Scientifique (CNRS) which supports this study.

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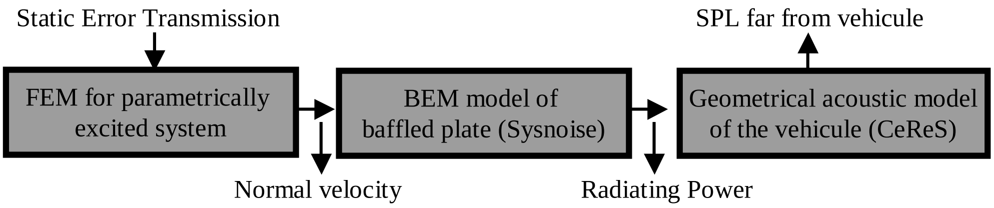

Figure 3: Diagram in three steps: Finite Element Method for the vibrational velocity, Boundary Element Method for the normal intensity and geometrical acoustics for the Sound Pressure Level.

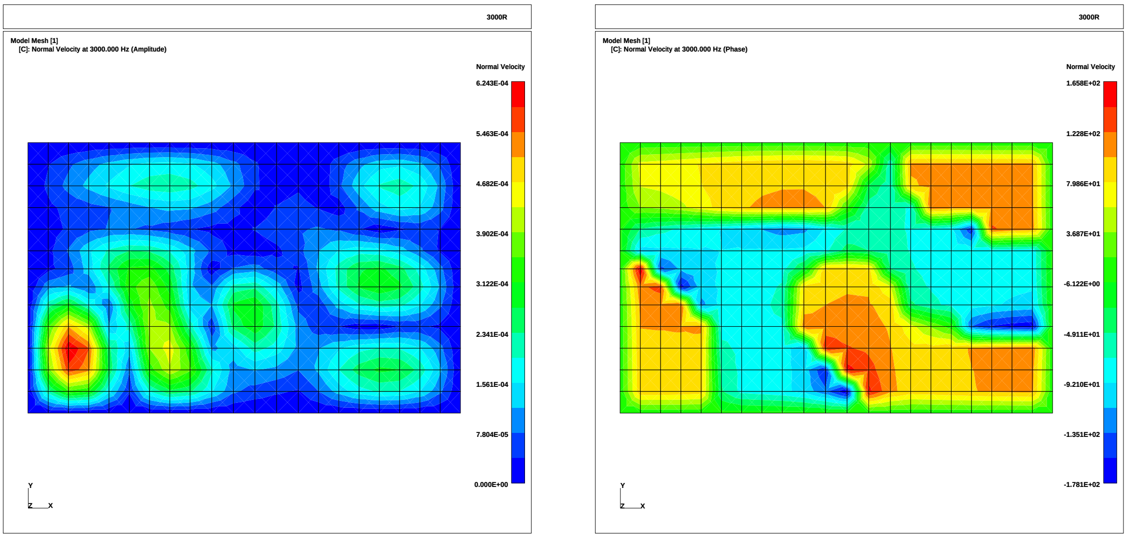

Figure 4: Magnitude and phase of normal velocity over the stiffened plate. The frequency is 3000 Hz and the plate is simply supported.

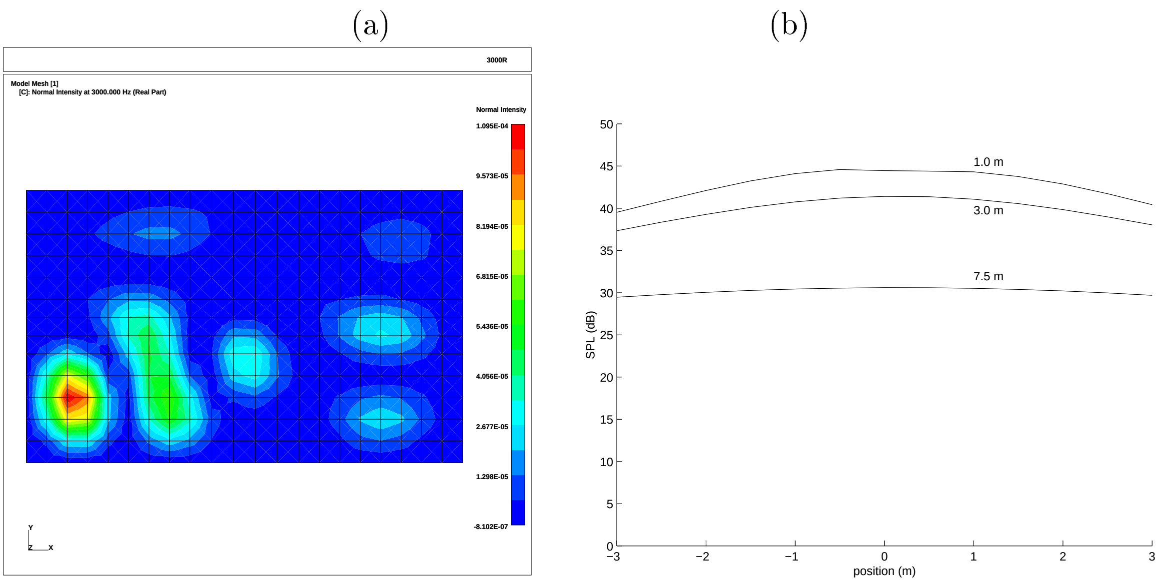

Figure 5: (a) Intensity over the stiffened plate assumed to be baffled. (b): SPL from back to front of the vehicule at 1 m, 3 m and 7.5 m.