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RADIATIVE TRANSFER EQUATION FOR TIME-REVERBERTATION OF CONCERT HALLS

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Abstract

This paper is concerned with the solving of the radiative transfer equation for the determination of time-reverberation in rooms. This method sometimes called the radiosity method is based on an analogy with the radiative heat transfer in thermics. It leads to an integral equation which can be solved with an iterative algorithm. A software (CeReS) has been designed to achieve this task. The present study is focused on the application of this method for numerical simulation of the auditorium Maurice Ravel in Lyon, France. A large acoustical model of the concert hall has been developped with CeReS. Results of numerical simulations have been compared with some measurements.

INTRODUCTION

For many years, some methods intended to assess the time-reverberation in rooms beyond the validity domain of Sabine's formula, have been investigated. Among them, the so-called radiosity method [1] applies for diffuse fields as well as for largely non-diffuse fields.

The radiosity method, based on the radiative transfer equation, is originally limited to the diffuse reflection (Lambert's law) although some extensions have been proposed for specularly reflecting surfaces [2, 3, 4]. The method also enables to determine the repartition of energy inside systems [5, 6] and, further, has been extended to vibroacoustics [7].

This method firstly developped by Kuttruff [8], has been deeply studied by Miles [9] who proved the uniqueness of the time-reverberation. Some algorithms to determine it have been proposed.

The purpose of this paper is to apply solve the radiative transfer equation for complex geometries with a large number of degree of freedom. It is shown that a simplified algorithm can save much CPU time compared with other algorithms. However, it leads to some limitations which are emphasized. Some results are presented for the concert hall Maurice Ravel in Lyon, France.

THEORETICAL METHOD

Let consider a room Ω of volume V enclosed by a surface Γ of area S and fullfilled with an acoustical fluid with a sound speed c and an attenuation factor m. The surface is assumed to be absorbing with a coefficient α defined as the ratio of reflected power over incident power and a reflection coefficient $\tau = 1 - \alpha$.

The reflection of energy is taken into account by some equivalent sources of magnitude $\sigma(\mathbf{q}, t)$ distributed over the surface Γ. At any point $\mathbf{p}$ at a distance R from $\mathbf{q}$ in direction θ, the radiative intensity is,

$$I(\mathbf{p}, t) = \sigma(\mathbf{q}, t - R/c) \cos\theta \frac{e^{-mR}}{4\pi R^2}. \tag{1}$$

since the source is assumed to radiate energy following the cosine Lambert's law. R/c is the duration for the energy propagates from $\mathbf{q}$ to $\mathbf{p}$. The incident power per unit surface when $\mathbf{p} \in \Gamma$ is $I(\mathbf{p}, t)\cos\varphi$ where φ is the incidence angle. In a second hand, the power emitted from $\mathbf{p}$ in all directions is given by integrating the intensity over a small sphere surrounding the source. It yields, $\sigma(\mathbf{p}, t)/4$. Now, at any point $\mathbf{p} \in \Gamma$, the energy balance reads $P_{\mathrm{refl}} = \tau P_{\mathrm{inc}}$ where P_{refl} is the reflected power (emitted from the equivalent source) and P_{inc} is the power incident from all other equivalent sources. Substituting the previous equations, the energy balance is

$$\sigma(\mathbf{p}, t) = \int_\Gamma \sigma\left(\mathbf{q}, t - r/c\right) \tau(\mathbf{p}) K(\mathbf{p}, \mathbf{q}) \mathrm{d}\Gamma(\mathbf{q}), \tag{2}$$

where,

$$K(\mathbf{p}, \mathbf{q}) = \cos\theta \cos\varphi \frac{e^{-mR}}{\pi R^2}. \tag{3}$$

Until now, it has been tacitly assumed that the domain Ω is convex, that is all points $\mathbf{q} \in \Gamma$ are viewed from the point $\mathbf{p}$. But, in the general case where some obstacles may lie inside Ω, it must be considered that some points $\mathbf{q}$ cannot contribute to the incident power at $\mathbf{p}$. The function K in equation (3) must be multiplied by the visibility function V defined by $V(\mathbf{p}, \mathbf{q}) = 1$ if $\mathbf{q}$ is viewed from $\mathbf{p}$ and $V(\mathbf{p}, \mathbf{q}) = 0$ otherwise.

Now, after the source is switched off, the decay of energy follows an exponential law $\sigma(\mathbf{p}, t) = \sigma(\mathbf{p})e^{-\lambda t}$ where $1/\lambda$ is the decay rate of energy. Thus,

$$\sigma(\mathbf{p}) = \int_\Gamma \sigma\left(\mathbf{q}\right) e^{\lambda \frac{R}{c}} \tau(\mathbf{p}) K(\mathbf{p}, \mathbf{q}) \mathrm{d}\Gamma(\mathbf{q}) \tag{4}$$

This is the radiative transfer equation for time-reverberation. The problem is now to determine the constant λ which allows the existence for a non negative function

σ. It is convenient to introduce the operator T_λ defined by $\mathrm{T}_\lambda\sigma = \int_\Gamma \sigma e^{\lambda R/c}\tau K \mathrm{d}\Gamma$ which maps $\mathrm{L}^2(\Gamma)$ into itself. Equation (4) then reads $(\mathrm{Id} - \mathrm{T}_\lambda)\sigma = 0$.

The uniqueness of λ follows from the variations of the real-valued function [9], $\lambda \longmapsto (\mathrm{Id} - \mathrm{T}_\lambda)\sigma(\mathbf{p})$ for any fixed point $\mathbf{p}$ whose derivative is

$$\lambda \longmapsto \int_\Gamma \sigma(\mathbf{q}) \frac{R}{c} e^{\lambda \frac{R}{c}} \tau(\mathbf{p}) K(\mathbf{p}, \mathbf{q}) \mathrm{d}\Gamma(\mathbf{q}) \tag{5}$$

which is a negative function since $\sigma, \tau, K \geq 0$. The former function is then decreasing and thus admits at most one zero.

SOME ALGORITHMS

The simpler method for solving equation (4) was proposed by Gerlach and Mellert [10]. The distance R is replaced by the average value $< R >= 4V/S$. The constant factor $e^{-\lambda<R>/c}$ is then an eigenvalue of the operator T_0 which is determined in one step by classical algorithms.

The second method proposed by Gilbert [11] assumes that the attenuation factor is zero and then $\int K d\Gamma = 1$ at any point. The factor λ is therefore solution of

$$\lambda = \frac{\int \alpha\sigma/\tau \mathrm{d}\Gamma}{\int\int \sigma K \left(e^{\lambda r/c} - 1\right)/\lambda \mathrm{d}\Gamma^2} \tag{6}$$

The algorithm is,

1. start with some realistic values for σ_0 and λ_0,
2. $\sigma_n = \mathrm{T}_{\lambda_{n-1}}\sigma_{n-1}$,
3. λ_n is computed from σ_{n-1}, λ_{n-1} with equation (6).

The third algorithm is proposed by Kuttruff [12]. The algorithm is as follows,

1. start with some realistic values for σ_0 and λ_0,
2. $\sigma'_n = \mathrm{T}_{\lambda_{n-1}}\sigma_{n-1}$ and $\sigma_n = \sigma'_n / \parallel \sigma'_n \parallel$,
3. $\lambda_n = \lambda_{n-1} - cS/4V \times \ln \parallel \sigma'_n \parallel$.

Finally, the method proposed in this text does not require the determination of the function σ.

1. start with a realistic value λ_0 (from Sabine's formula),
2. find a root for $\det(\mathrm{Id} - \mathrm{T}_\lambda)$ around λ_0 with any standard algorithm,
3. check that $\sigma \geq 0$.

The last step is necessary because in general the equation $\det(\mathrm{Id} - \mathrm{T}_\lambda) = 0$ has several solutions. But only one admits a non-negative function σ verifying equation (4). This is an important limitation of the present algorithm. It runs faster provided that the good root is found. A software named CeReS has been designed to solve equation (4) with this algorithm for complex structures.

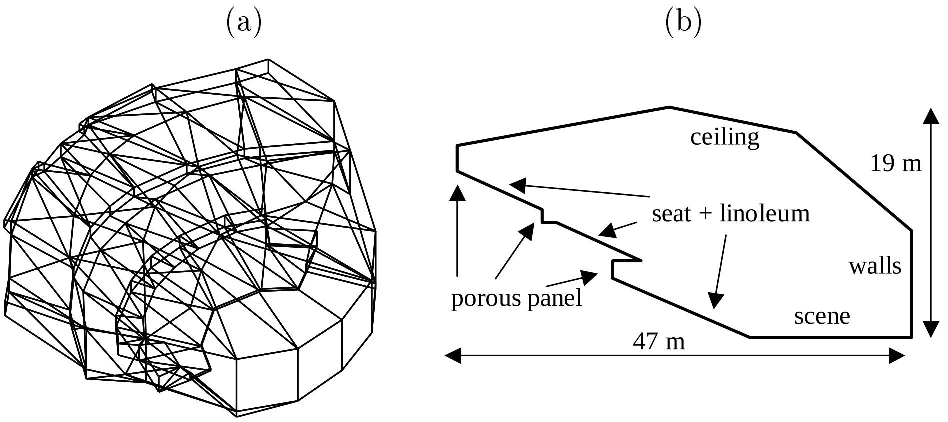

Figure 1: View of the geometry of the concert hall Maurice Ravel.

	125 Hz	250 Hz	500 Hz	1kHz
Ceiling	0.13	0.13	0.10	0.10
Seat	0.27	0.27	0.23	0.27
Scene	0.10	0.07	0.05	0.05
Linoleum	0.02	0.02	0.03	0.03
Wall	0.40	0.30	0.20	0.10
Porous panel	0.13	0.73	0.85	0.72

Table 1: Table of absorption coefficient for the auditorium.

CONCERT HALL MAURICE RAVEL

The concert hall Maurice Ravel in Lyon has a volume of 22000 m^3. The total area of walls, floor and ceiling is 6000 m^2 with a mean absorption coefficient $\bar{\alpha} = 0.2$. The acoustical constant is $R = 1500$ m^2. The time-reverberation for the empty auditorium is about 3 s over the audible range.

The auditorium has been modelled by CeReS with the geometry shown in Figure 1. About 210 faces were necessary for a total amount of 1000 triangles of meshing. The enclosure is not convex and, thus, the visibility function must be carefully computed. Six different absorption coefficients have been introduced in the model for seats, walls (wood), floor (linoleum), scene (hard wood), ceiling and back walls (porous panel). Their values (Table 1) were taken from some tables available in the literature (in situ measurements have not yet been carried out).

In Table 2 are the time reverberations determined from Sabine's formula, the radiative transfer equation and some measurements (empty auditorium). But it is significant to observe that the difference between Sabine's formula results and the ones of radiative transfer are about 10%. Unfortunatly, the measurements cannot

Tr	125 Hz	250 Hz	500 Hz	1kHz
Sabine	3.3 s	2.6 s	2.8 s	2.8 s
Radiosity	2.7 s	2.4 s	2.6 s	2.5 s
Measurements	3.0 s	2.9 s	2.6 s	2.5 s

Table 2: Table of time-reverberation (empty room).

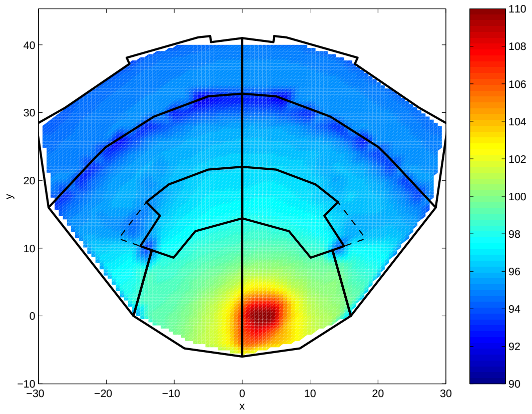

Figure 2: Predicted SPL inside the concert hall (empty room) at 1 kHz.

decide between Sabine's formula and the radiative transfer equation since the uncertainty on the absorption coefficients introduced in the model, results in a variability greater than 10% of reverberation time.

In Figure 2 is shown the repartition of sound inside the hall. Although the uniformity of sound may be considered as sufficient to ensure an equal quality of listening, the sound is not totally diffuse. This is probably the reason of the discrepancy betwwen Sabine's formula and the radiative transfer equation.

CONCLUSION

A numerical scheme for solving the radiative transfer equation has been proposed in this paper. The time-reverberation of rooms is more accurately determined than by applying Sabine's formula. Furthermore, this algorithm runs faster than other algorithms proposed in the literature, and thus, is applicable for large rooms with complex geometry. However, the uniqueness of time-reverberation is no longer ensured. A first determination of time-reverberation with Sabine's formula is required to assess an interval where the time-reverberation is expected. This is a limitation of the present approach.

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