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FROM NON DIFFUSE FIELD TO STATISTICAL ENERGY ANALYSIS

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ABSTRACT

This paper is concerned with the derivation of SEA equations from structural ray equations. Rays are assumed to be uncorrelated leading to the additivity of energy. Inside all subsystems, the energy density is the sum of a direct field from driving forces, a reflected field from boundary and a transmitted field from adjacent subsystems. Assuming a "rain-on-the-roof" excitation and a compact shape for subsystems, actual and fictitious sources on boundary are found to be constant. Furthermore, if the attenuation of rays during a mean free path (normalized attenuation factor) is light, the field becomes diffuse i.e. homogeneous and isotropic. The net exchanged power between two adjacent subsystems is then proportional to the difference of energy densities and therefore, to the difference of modal energies. The derived proportionality coefficient is consistent with the well-known formula for coupling loss factor in terms of transmittivity. These results are illustrated by a numerical simulation for a multi-plate system. Finally, a discussion on the validity of SEA and ray theory is proposed and the normalized attenuation factor turns out to be an appropriate indicator for diffuse field.

1 INTRODUCTION

The description of vibration fields in structures by means of energy quantities is a common idea in high frequency range. Statistical Energy Analysis (SEA) [1] is certainly the most widely spread theory in this field and its success attests the efficiency of such an approach. The statistical point of view adopted in SEA, assumes that the vibration fields are in thermal equilibrium. This is the so-called diffuse field assumption. This statistical mechanics approach leads, up to now, to new and non trivial results [2, 3].

Naturally, the question arises whether it is possible to predict the distribution of energy inside each sub-systems when the field is non diffuse or, in other words, when the thermal equilibrium is not reached. Several methods have been proposed [4-7] always based on the energy approach.

The purpose of this paper is to investigate the transition between non equilibrium to equilibrium state. We start from the uncorrelated ray theory exposed in Ref. [8] and valid for diffuse as well as non diffuse fields. Then, by introducing the diffuse field assumption, we get SEA and a condition for the validity of SEA.

2 FROM NON DIFFUSE TO DIFFUSE FIELD

When rays are uncorrelated, their energies can be simply added. Since the energy of a single ray is $e^{-mR}/2\pi c_i R$ where R is the source-receiver distance, c_i the group speed and $m=\eta\omega/c_i$ the attenuation factor, the energy at any point in a subsystem i is given by a radiosity integral [7],

$$W_i(\mathbf{r}) = \int_{\Omega_i} \rho_i(\mathbf{s}) \frac{e^{-mR}}{2\pi c_i R} d\Omega_{\mathbf{s}} + \int_{\Gamma_i} \sigma_i(\mathbf{q}) \cos\theta \frac{e^{-mR}}{2\pi c_i R} d\Gamma_{\mathbf{q}} , \tag{1}$$

where ρ_i is the power density of mechanical sources and σ_i is the power density of rays reflected by the boundary Γ_i of the domain Ω_i. The term $\cos\theta$ in the second integral means that the boundary sources σ_i radiate energy in accordance with Lambert's law. At the interface between two plates, the power per unit length of rays being transmitted from subsystem i to subsystem j is,

$$\frac{dP_{i\to j}}{dL} = \int_{\Omega_j} \tau_{ij}(\varphi) \rho_i(\mathbf{s}) \frac{e^{-mR}}{2\pi R} \cos\varphi d\Omega_{\mathbf{s}} + \int_{\Gamma_i} \tau_{ij}(\varphi) \sigma_i(\mathbf{q}) \cos\theta \frac{e^{-mR}}{2\pi R} \cos\varphi d\Gamma_{\mathbf{q}} , \tag{2}$$

where φ is the incidence angle on the boundary and τ_{ij} is the transmission efficiency from subsystem i to subsystem j at incidence φ.

Diffuse field requires a "rain-on-the-roof" excitation ρ_i=cste and that the density of reflected energy is also constant σ_i=cste. Eqs. (1), (2) then become,

$$W_i = \rho_i \frac{l}{2c_i} + \frac{\sigma_i}{c_i} \quad \text{and} \quad P_{i\to j} = L\frac{\overline{\tau}_{ij}}{\pi}\left(\rho_i l + \sigma_i\right), \tag{3}$$

where l is the mean free path of the plate, L the length of the coupling and $\overline{\tau}_{ji}$ the mean transmission efficiency. The proportionality between the second terms is,

$$P_{i\to j} = Lc_i \frac{\overline{\tau}_{ij}}{\pi}\frac{E_i}{S_i}, \tag{4}$$

where $E_i=W_iS_i$ is the total vibrational energy of subsystem i with area S_i. This is the classical SEA equation with the coupling loss factor,

$$\eta_{ij} = L\cdot\frac{c\tau_{ij}}{\pi\omega S_i}, \tag{5}$$

Thus, SEA applies provided that $\rho_i l<<\sigma_i$ or, in other words, when the direct field is negligible compared with the reverberent field. Another way to write this condition is to introduce the normalized attenuation factor $\underline{m}=\eta\omega l/c_i$. The condition is then $\underline{m}<<1$.

3 NUMERICAL RESULTS

A numerical simulation is proposed on the structure shown in Fig. 1. The structure is made of seven identical aluminium plate, density ρ=2700 kg/m^3, Young's modulus E=71 GPa, Poisson's coefficient ν=0.3, width and length of plates L=1 m, thickness h=1 mm, damping loss factors η=0.1%, 1% and 10% (three cases). The assembling (4 T-junctions and 2 L-junctions) is assumed to be perfect, *i.e.* the continuity of transverse deflection, rotation and the balance of forces and moments apply. Plate 1 is submitted to 16 external point forces whose positions are shown in Fig. 1 (small hammers). The receiver points are located at the centre of each plate.

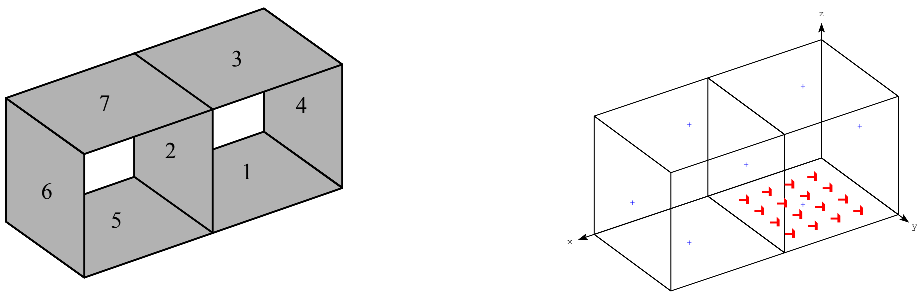

Fig. 1. The seven plate structure. Driving points (small hammers) and receiver points (+).

Three calculations have been carried out. The first one is the reference calculation (direct numerical simulation). It consists in the solving of the governing equations. In-plane motions are neglected and therefore, the governing equation reduces to the single Love's equation for

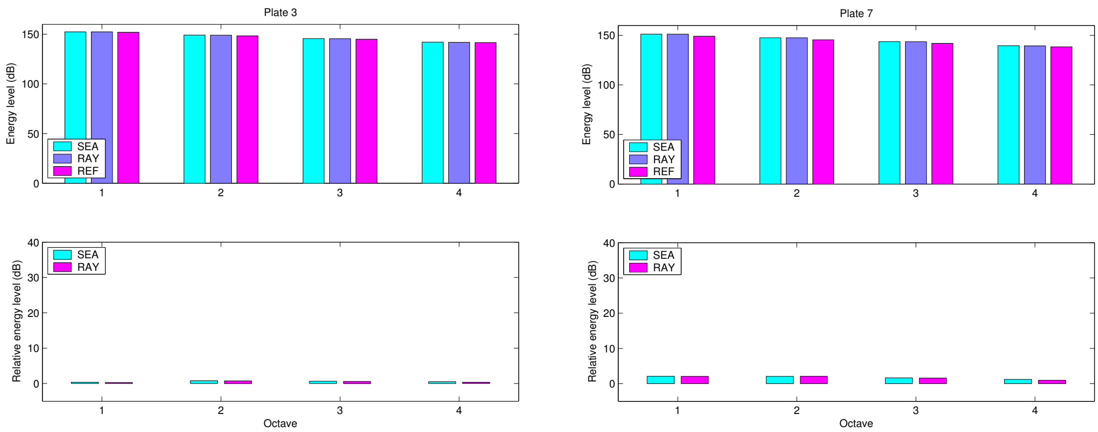

Fig. 2. Vibrational response of plates 3 and 4. for damping loss factor η=0.1%.Comparison between SEA, uncorrelated rays (RAY) and reference calculation (REF) for the four octave bands from 1 kHz to 8 kHz. The errors between SEA/REF and REF in dB are shown in the bottom bar diagrams.

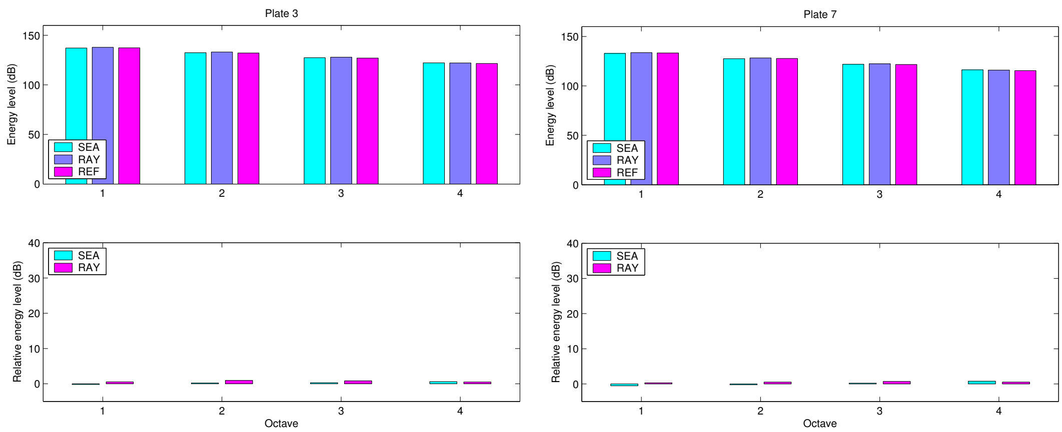

Fig. 3. Idem for damping loss factor η=1%.

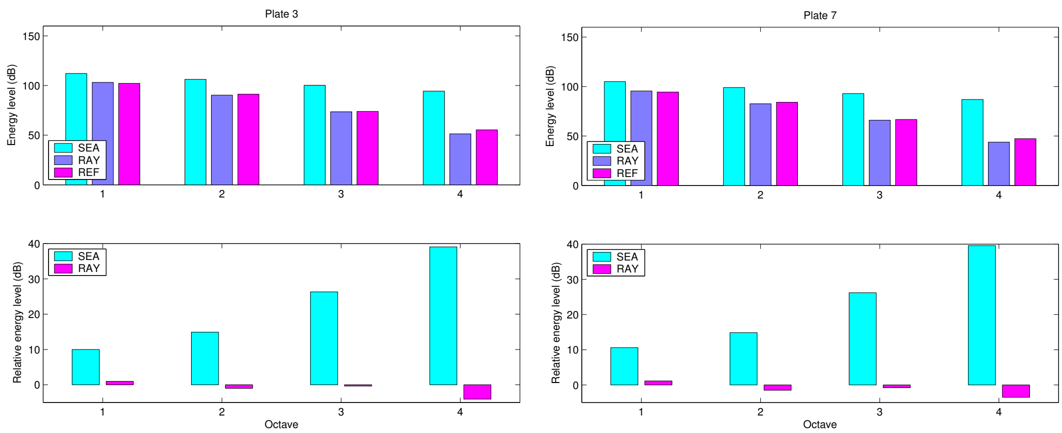

Fig. 4. Idem for damping loss factor η=10%.

transverse deflection. The coupling conditions at L-junctions are identical with the case of two coplanar plates with a simply supported common edge. The calculation is done from 707 Hz up to 11 312 Hz (4 octaves), one step per Hz. Each driving point applies a force F=1 N (peak value). As the forces are assumed to be uncorrelated, the governing equations are solved separately for the sixteen loading cases. The energies of these individual excitations are simply summed. The final result of this direct numerical simulation is the RMS-response over each octave band. The second method is SEA. The coupling loss factors are defined by Eq. (5). This coupling loss factor, although it is not the most efficient one, has been adopted in order to be comparable with the third method based on the uncorrelated ray integral.

Table 1. Modal overlap M and normalized attenuation factor $\underline{m}$ versus frequency f and damping loss factor η.

M / $\underline{m}$	1 kHz	2 kHz	4 kHz	8 kHz
η=0.1%	0.3 / 0.02	0.6 / 0.03	1.2 / 0.05	2.4 / 0.07
η=1%	3 / 0.2	6 / 0.3	12 / 0.5	24 / 0.7
η=10%	30 / 2	60 / 3	120 / 5	240 / 7

In Table 1 are shown the modal overlap and the normalized attenuation factor for the four octave bands and the three damping loss factors. Results of simulations are shown in Figs. 2-4. Vibration levels of the three methods are presented in the upper bar diagrams whereas the differences of SEA (SEA) and ray results (RAY) with the direct numerical simulation (REF) are shown in dB in the lower bar diagrams. A good agreement between the three methods is met for the cases η=0.1% and 1% for which the normalized attenuation factor is less than one (Table 1). But some significant differences appear when η=10% between SEA and the direct numerical simulation although the RAY calculation still well works. This is due the large value of the normalized attenuation factor which emphasizes that the field becomes non diffuse.

4 VALIDITY DOMAIN

The validity domain of the energy methods may be assessed. Both methods require a large number of modes N. But, the modal density n is constant in frequency for plates and since the bandwidth is doubled for successive octave bands, it yields $N \propto f$ where f is the central frequency. The limit N=cste is then a vertical line in the η,f-plane of Fig. 5. The modal overlap is $M=\eta\omega n$ and the boundary of the domain M>1 is therefore the hyperbolic line $\eta=1/(2\pi nf)$ which is the bottom solid line of Fig. 5. Finally, the group speed c is proportional to $c \propto f^{1/2}$. It results in a normalized attenuation factor $\underline{m} \propto \eta f^{1/2}$. The upper solid line of Fig. 5 is therefore $\eta = a/f^{1/2}$ where a is a proportionality constant. The validity domain of the energy exchanges is N>cste and M>1 and is thus quater plane shown in Fig. 5. The validity domain of SEA is limited by N>cste, M>1 and $\underline{m}$<1 and is therefore the half-strip shown in Fig. 5. The only case located within the strip is η=1% where the three methods well agree.

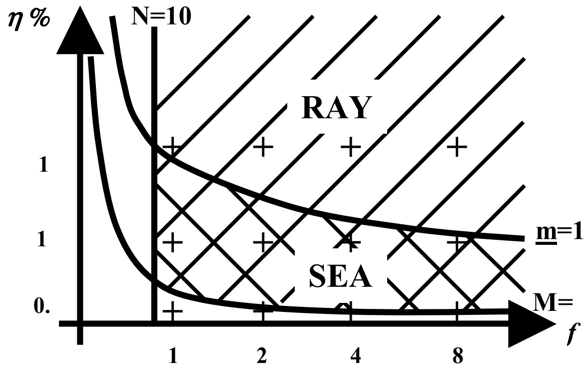

Fig. 5. Validity domain of SEA and uncorrelated rays in the frequency – damping loss factor plane.(\\), SEA domain defined by N>>1, M>1, and m<1. (//), Ray domain defined by N>>1 and M>1. (+), position of the twelve calculations of Figs. 2, 3, 4.

5 CONCLUSIONS

The present paper has investigated the way from non diffuse to diffuse field in structures made of assembled plates. It has been shown that in condition of diffuse field, SEA well applies. It has also be shown that the ray theory well converges to SEA when the field evolves from a non diffuse state to a diffuse state. Thus, in conclusion, in addition to the criteria of large number of modes N>>1 and a large modal overlap M>1 which ensure that field is in high frequencies, it appears that the normalized attenuation factor $\underline{m}=\eta\omega l/c$ must be lower than one, $\underline{m}$<1. This additional condition is important since if the damping loss factor is maintained constant, the field becomes non diffuse when the frequency increases.

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