Asian Journal of Scientific Research 4 (1): 28-41, 2011
ISSN 1992-1454 / DOI: 10.3923/ajsr.2011.28.41

Vehicle Stopping Distance by Means of Suspensions Control

[1]T.R. Ori, [1]P. Gbaha, [1]O. Asseu and [2]A. Le Bot
[1]Institut National Polytechnique Houphouët Boigny, Côte d'Ivoire
[2]Laboratoire de Tribologie et Dynamique des Systèmes-UMR 5513, Ecole Centrale de Lyon-France 36, av. Guy de Collongue-69131 Ecully, France

Corresponding Author: P. Gbaha, INP-HB, BP 1093 Yamoussoukro, Côte d'Ivoire Tel: (+225) 07 86 73 38

ABSTRACT
This study addresses a new concept of intelligent braking functionality. The new concept proposed is based on the use of energy transfer from suspension behaviour. It tends, in fact, to reduce the energy needs for braking command using the overall vehicle's dynamics. A simple 2-D model is herein described in order to show the feasibility of the new concept. A new active control strategy based on an LQ (Linear Quadratic) type controller is then built. This control law takes into account the braking abilities peculiarities. Hence, for acting directly on the suspension devises, one can improve notably braking performances. The numerical computational results confirm this finding and provide further ideas improvement and extension.

Key words: Braking system, friction force, vibratory comfort, suspension, active control

INTRODUCTION
The main objective of this study is to decrease the stopping distance of a vehicle in straight line by using the active suspensions.

Active suspensions were introduced at the beginning into the transport to improve vibratory comfort. Several systems have been proposed on secondary suspension. This work proposes to control the normal forces, using linear quadratic law, exerted by the wheels on the road. It is the vertical acceleration of axle in a short time which creates these normal forces during braking phase.

The effectiveness of our system is being seen by the difference of vehicle's stopping distance between control and without control of suspensions.

Here, we present the dynamic equations model used for numerical simulation. This model is a half-car-model combined with a Pacejka Wheel's model (Pacejka, 2006) which gives an empirical relation between the forces of friction exerted on the contact surface, the normal force and the slip ratio (Frendo *et al*., 2006).

To evaluate the performances of brakes of a vehicle in a straight line, we do not take into account the movements of the vehicle in the side plan in this study. Hence, the equations of the model will be established by taking into account only of the vertical and longitudinal dynamics of the vehicle.

The braking system simulation includes the tire-road interface using the slip ratio (Alleyne, 1997). We observe in the horizontal plane, the vehicle stopping distance without action on the vertical normal force.

PRESENTATION OF NUMERICAL MODEL

Half-car-model: For the sake of clarity, a simple 2-D model representing half-car behaviour is used. It aims at validating the approach in this study. The state space equation is shown for the force and moments equilibrium of the half car model (Fig. 1).

This model of vehicle is composed of three rigid bodies (Baslamisli *et al*., 2009). The suspended mass represents the rigid body and the two unsprung masses represent the front and rear axles. This model includes 9 (DOF) degrees of freedom: three degrees of freedom for the rigid body (x, z, θ), two degrees of freedom for each unsprung mass (x, z), one degree of freedom for the rotation (w) of each wheel.

In fact using the above definitions, the dynamics of the half car model can be represented by the following equations. During braking phase in horizontal plane, movement is given by the Eq. 1.

$$m_s \ddot{x} = -f_{av} - f_{ar} \tag{1}$$

where, f_{av} and f_{ar} represent respectively the front and rear friction forces of vehicle between the contact surface and the road. m_s represents the sum of the three rigid masses.

The oscillatory movements of the vehicle's rigid mass are given, in the vertical plane by the Eq. 2.

$$m_s \ddot{z} = f_v + f_r \tag{2}$$

with

$$\begin{aligned} f_v &= -k_{sv}(z_{sv} - z_{rv}) - b_{sv}(\dot{z}_{sv} - \dot{z}_{rv}) + u_v \\ f_r &= -k_{sr}(z_{sr} - z_{rr}) - b_{sr}(\dot{z}_{sr} - \dot{z}_{rr}) + u_r \end{aligned} \tag{3}$$

The rotation of the rigid mass caused by a load transfer from rear to front is given by the Eq. 4.

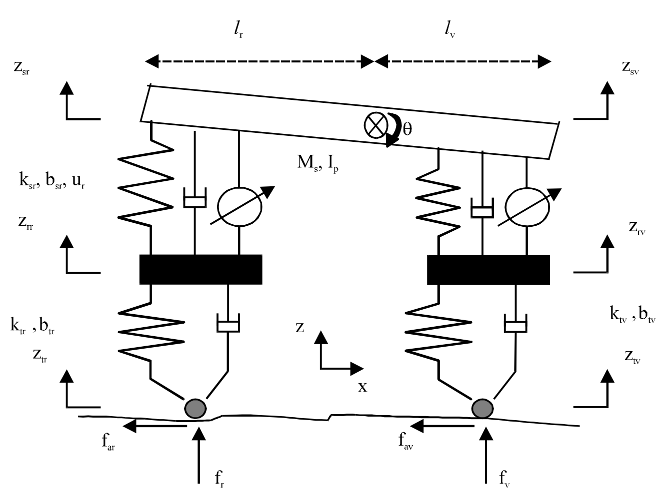

Fig. 1: Half car demonstrative model

$$I_p\ddot{\theta} = f_v.l_v - f_r.l_r - f_{av}(z_{sv} - z_{rv} + h) - f_{ar}(z_{sr} - z_{rr} + h) \quad (4)$$

The deviation angular of the rigid mass caused by the front and rear suspension roughly equals:

$$\begin{aligned} z_{sv} &= z + l_v.\theta \\ z_{sr} &= z - l_r.\theta \end{aligned} \quad (5)$$

These equations of half car model are used with wheel model equations to improve vehicle stopping distance.

Wheel model: On wheel level, the movement equation (Ori, 2001) is given as:

$$J.\dot{w} = \text{friction force}.r - T_b \quad (6)$$

The friction forces depend on the wheel slip ratio which is defined in a following way.

$$\lambda = \begin{cases} \dfrac{v - w.r}{v} & \text{si} \quad w.r < v \\ \dfrac{w.r - v}{w.r} & \text{si} \quad w.r \geq v \end{cases} \quad (7)$$

In this approach, only vehicle braking force is considered. The relationship between wheel slip ratio and longitudinal friction force is usually given by an empirical formula (Frendo *et al.*, 2006). In this study, we use Pacejka's wheel model. This model gives the friction force (f_a) when we know the normal force (F_z) and the slip ratio. The Pacejka's formula is:

$$f_a = D.\sin[C.\tan^{-1}\{B.\lambda - E.(B.\lambda - \tan^{-1}(B.\lambda))\}] \quad (8)$$

The formula coefficients (B, D and E) depend on normal force. The coefficients that are used in the model and simulation are given as:

$$C = 1.8 \qquad \text{(Shape factor)}$$

$$D = a_1.F_z^2 + a_2.F_z \qquad \text{(Peak factor)}$$

$$B = \frac{a_3.F_z^2 + a_4.F_z}{C.D.\exp(a_5.F_z)} \qquad \text{(Stiffness factor)}$$

$$E = a_6.F_z^2 + a_7.F_z + a_8 \qquad \text{(Curvature factor)}$$

$$\begin{aligned} &a_1 = -21.3 \quad a_2 = 744.0 \quad a_3 = 49.6 \quad a_4 = 226.0 \\ &a_5 = 0.3 \quad a_6 = -0.006 \quad a_7 = 0.056 \quad a_8 = 0.486 \end{aligned}$$

These coefficients a_i are defined experimentally (Alleyne, 1997).

These equations of numerical model are integrated in a simulation environment. This model was developed at 2000 by the structures and systems dynamics team of the Ecole Centrale de Lyon. The law of control was improved in 2006 to the Institute National Polytechnique Houphouët Boigny de Yamoussoukro, Côte d'ivoire.

Control laws developments: The law of action that we will formulate is based on control LQ, i.e., the minimization of a quadratic criterion for a linear dynamic system (Zhang and Alleyne, 2005). We control at every pitch of time the normal force in the vertical plane to decrease the vehicle stopping distance (Ori, 2001). Control consist in reducing the quadratic displacement of the boogie in the vertical plane and reduce the deviation angular of the rigid mass caused by the load transfer from rear to front of vehicle (Ori *et al.*, 2007). The objective is to find a control law as U = -G.Z. We define a criterion to minimize.

$$J = \lim_{T\to\infty} \frac{1}{T} E\int_0^T \left(Z^T.Q.Z + U^T.R.U + 2.X^T.N.U\right)dt$$

In this study, we use:

$$J = \lim_{T\to\infty} \frac{1}{T} E\int_0^T \left(\rho_v.u_v^2 + \rho_r.u_r^2 + q_1.(\ddot{z}_{rf})^2 + q_2.(\ddot{z}_{rr})^2 + q_3.(z_{sf} - z_{rf})^2 + q_4.(z_{sr} - z_{rr})^2\right) dt$$

We resolve the Riccati's equation $A^T.P(t) + P(t).A + Q - P(t).B_1.R^{-1}.B_1^T.P(t) = 0$ to find the expression of $G = R^{-1}.B_1^T.P(t)$.

P, Q and R are defined, symmetric and positives matrices. These are control forces u_f and u_r which act as the suspensions in the vertical plane.

$$\begin{bmatrix} u_v \\ u_r \end{bmatrix} = -R^{-1}.\, B_1^T.P(t).Z$$

The details of the establishment of this law of control are in appendix A. The synthesis of the law of control is directed by the various bodies of the system in order to provide the function of suspension as well as possible.

Resolution of the system: The curves of the theoretical results are obtained by a numerical resolution of the equations of the system and law of control. The equations of the system are put in the form of differential equations, easy to integrate if the initial term is known. The method of resolution used is of type Runge Kutta to single step.

The research of the law coefficients of control is done by the use of software MATLAB. The coefficients are determined by the function LQ (Linear Quadratic) of software. This programme for the determination of the law coefficients is detailed in appendix B.

RESULTS

A vehicle of 730.0 kg on a normal way is used for simulation. The vehicle stopping distance with and without control is observed. The vertical displacement of rigid mass is also observed to show the lack of system incidence on the comfort.

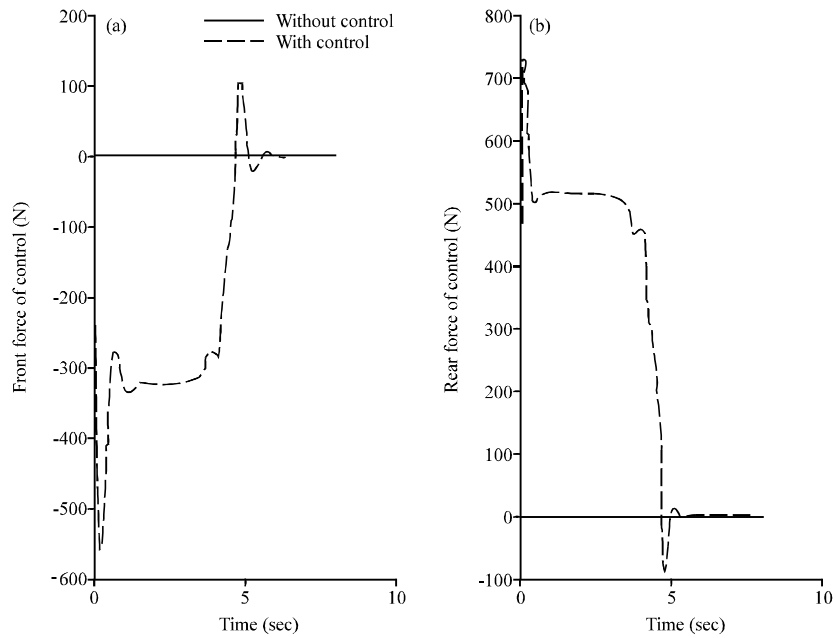

Fig. 2: Suspensions control force

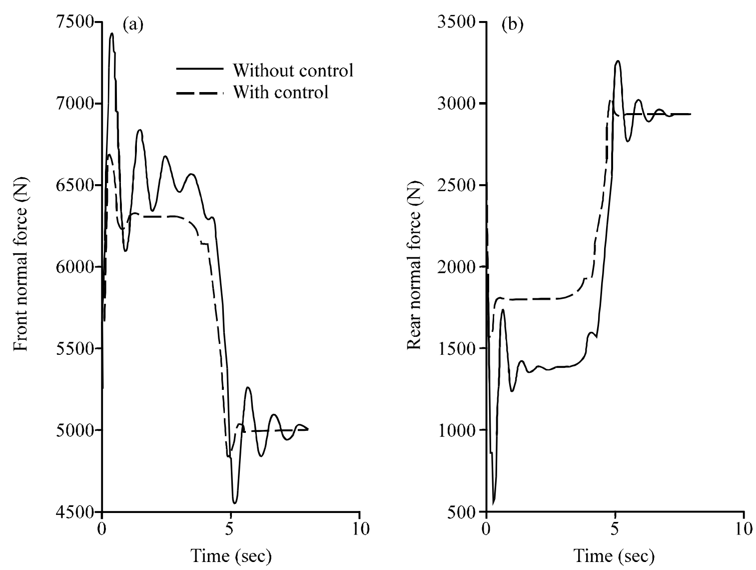

Fig. 3: Normal forces applied to the vehicle's wheel

This vehicle runs at 27.0 m sec^{-1} when the driver operates the footbrake pedal. It spends 4.3 sec to stop the car and it traverses 62 m when the suspensions are not controlled. Simulation starts when the driver operates the footbrake pedal and not the time of the perception of an obstacle.

The force of control for the front suspension is weak whereas on the rear suspension (Fig. 2a, b) the force of control is stabilized to 500 Newton during 4 sec before being cancelled at the end of braking. The normal force (Fig. 3a, b) on the vehicle with control oscillates and they are more stable than the vehicle without control.

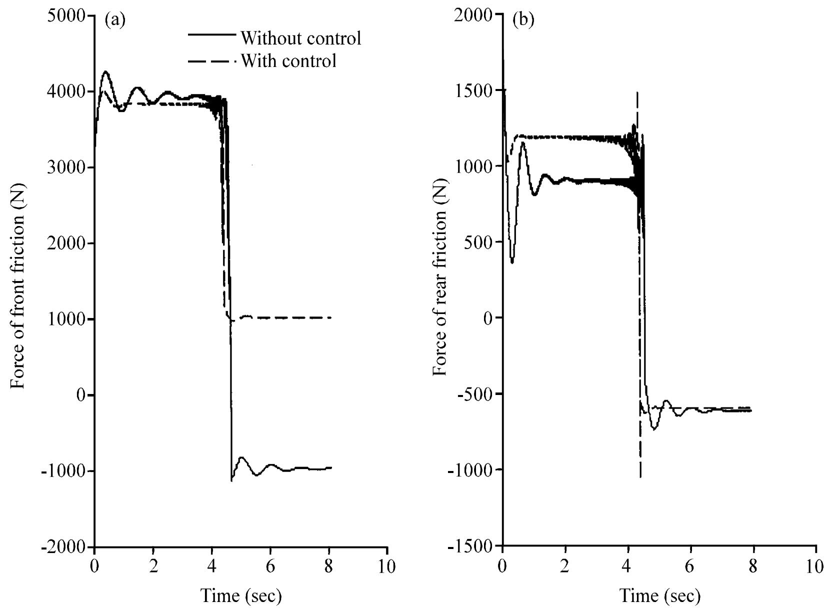

Fig. 4: Longitudinal friction forces between the wheel and the road

For the front force of friction, there is a weak difference between the vehicle with and without control. On the other hand, the difference of the front force of friction (Fig. 4a, b) is more significant than the one on the vehicle with control.

The front and rear displacements of rigid mass of the vehicle (Fig. 6) without control are more significant than displacements of the vehicle with control.

DISCUSSION

Many theoretical and experimental works are proposed for applications of optimal control theory for the active control of mechanical systems and structures vibrations. Several works, such as those of Barzamini *et al.* (2009) which overcome the problems of noise and disturbance used suspension system. Wu *et al.* (2004) describe the principle and application of Active Vibration Control (AVC) for reducing undesired small-amplitude vertical vibration in the driver's seat of a vehicle. The results presented in the study of Rashid *et al.* (2006) justifying that the semi-active suspension system can be effectively employed to the passenger vehicle with improved both ride comfort and steering stability. Baslamisli *et al.* (2009) studied the variation of the adhesion coefficient according to the speed to improve the vehicle handling during the significant curves phase. Ori *et al.* (2007) improve the road behaviour of the car in curves during braking phase.

Frendo *et al.* (2006) studied an ideal behaviour of vehicle to obtain the trajectory desired according to the steering torque and the longitudinal force. Zhang and Alleyne (2005) propose an experimental methodology to the active suspension based on prescribing a given displacement between sprung and unsprung masses. The law of control used in this study brings an improvement to those studied by Alleyne (1997).

Present study, unlike the others, proposes a method using the suspensions to reduce the braking distance. It is effective on the degraded road, contrary to the ABS (Anti-lock Braking System) which degrades braking on the roads of bad qualities.

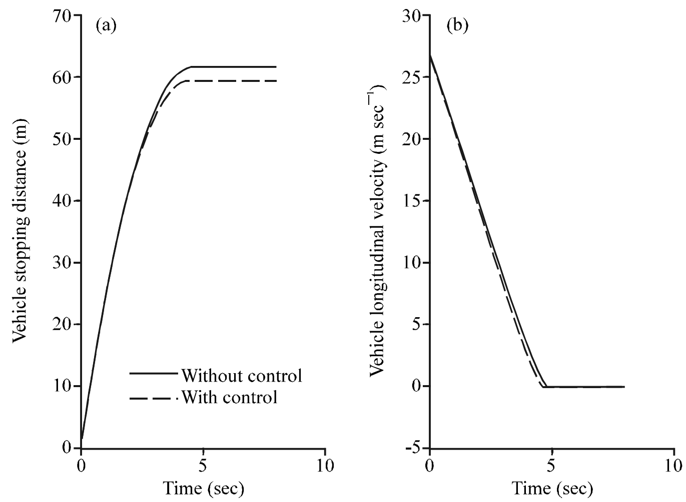

Fig. 5: Vehicle velocity and stopping distance

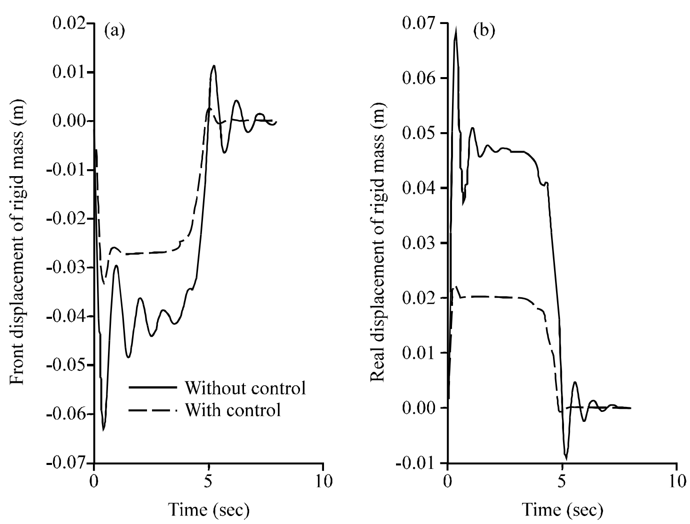

Fig. 6: Displacement of rigid mass

The control of the vertical forces (Fig. 2) provides an additional load on the rear wheels. It is not the case with the nose gear wheels which are charged, because of the load transfer from rear towards the front of vehicle.

That improves the normal force and the friction force at the rear of the vehicle, causing a reduction of the vehicle braking distance from 3 to 4%. An additional load on the wheels of the nose gear, which are in extreme cases of saturation, could prove to be dangerous for the safety of the passengers. It can make the operation difficult by blocking the saturated wheel or lead to the bursting of the wheel.

For significant speeds, the reduction of the braking distance will be significant. For a vehicle with an initial velocity to 36.11 m sec^{-1} (that is to say approximately 130.0 km h^{-})^{1}and which brakes in straight line, the control of the suspensions gives a reduction by 4 m on the braking distance which corresponds to a reduction of 3.65% (Fig. 5a, b).

The advantages on the braking distance would be greater if our law of control did not contain term X^TNU induced by the expressions of accelerations. This improvement of the angle of pitching thus involves a reduction in the load transfer from rear towards the front of vehicle and also a better passenger comfort (Fig. 6a, b).

These simulations have shown that braking in straight line, did not permit control to have many effects on the suspensions before the vehicle. The simulations, by removing the control of the suspensions at the front of vehicle, gave the same results as those observed in our study.

CONCLUSIONS

Our study has permitted us to investigate modelled and simulated active suspension to improve the vehicle stopping distance during the braking phase.

Our target margin satisfying results was limited. Nowadays indeed tyres are employed at around 95 % of their maximum efficiency. As tyres are designed to endure up to 110 % of their possibility, we can add a reasonable supplementary load.

From the general results of the optimization, an active command law on a nonlinear system was synthesized. This control law showed that it is possible to reduce by 3 to 4% the stopping distance of the vehicle (Fig. 5) while acting on the suspensions. But on the whole of the existing active suspensions, those which are likely to find an industrial application are those which are robust, reliable and least expensive.

APPENDIX

Appendix A: This model (Baslamisli *et al*., 2009) of vehicle is composed of three rigid bodies. The suspended mass represents the rigid body and the two unsprung masses represent the front and rear axles. This model includes 9 (DOF) degrees of freedom: three degrees of freedom for the rigid body (x, z, θ), two degrees of freedom for each unsprung mass (x, z), one degree of freedom for the rotation (w) of each wheel. The dynamic system can be expressed in the form:

In the vertical plane:

$$\dot{Z} = A_1 Z + B_1 U + B_2(Z).F + B_3 W \qquad (1)$$

with $A_1(8\times8)$, $B_1(8\times2)$, $B_2(8\times2)$ as a function Z and $B_3(8\times4)$.

To determine the law of control, we need only the matrices A_1 and B_1,

$$A_1 = \begin{bmatrix} 0 & 1 & 0 & -1 & 0 & 0 & 0 & 0 \\ -\alpha_1.k_{sf} & -\alpha_1.b_{sf} & 0 & \alpha_1.b_{sf} & -\alpha_2.k_{sr} & -\alpha_2.b_{sr} & 0 & \alpha_2.b_{sr} \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ -\frac{k_{sf}}{m_{us}} & -\frac{b_{sf}}{m_{us}} & -\frac{k_{tf}}{m_{us}} & \frac{(b_{sf}-b_{tf})}{m_{us}} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & -1 \\ -\alpha_2.k_{sf} & -\alpha_2.b_{sf} & 0 & \alpha_2.b_{sf} & -\alpha_3.k_{sr} & -\alpha_3.b_{sr} & 0 & \alpha_3.b_{sr} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & -\frac{k_{sr}}{m_{ur}} & -\frac{b_{sr}}{m_{ur}} & -\frac{k_{tr}}{m_{ur}} & \frac{(b_{sr}-b_{tr})}{m_{ur}} \end{bmatrix}$$

and

$$B_1 = \begin{bmatrix} 0 & 0 \\ \alpha_1 & \alpha_2 \\ 0 & 0 \\ \frac{1}{m_{us}} & 0 \\ 0 & 0 \\ \alpha_2 & \alpha_3 \\ 0 & 0 \\ 0 & \frac{1}{m_{ur}} \end{bmatrix}$$

With

$$\alpha_1 = \frac{1}{m_s} + \frac{l_f^2}{I_p};$$
$$\alpha_2 = \frac{1}{m_s} - \frac{l_f.l_r}{I_p};$$
$$\alpha_3 = \frac{1}{m_s} + \frac{l_r^2}{I_p};$$

In the horizontal plane:

$$\dot{X} = A_2 X + B_4 F + DT \qquad (2)$$

with $A_2(6\times6)$, $B_4(6\times2)$ and $D(6\times2)$.

The vectors X and Z are:

$$Z = \begin{bmatrix} z_{sv} - z_{rv} \\ \dot{z}_{sv} \\ z_{rv} - z_{tv} \\ \dot{z}_{rv} \\ z_{sr} - z_{rr} \\ \dot{z}_{sr} \\ z_{rr} - z_{tr} \\ \dot{z}_{rr} \end{bmatrix} \quad X = \begin{bmatrix} x \\ \dot{x} \\ w_{av} \\ \dot{w}_{av} \\ w_{ar} \\ \dot{w}_{ar} \end{bmatrix}$$

The choice of vectors was made for a suitable search for our control law, which constitutes an important phase of this study.

Expression of the criterion: The control law that we formulate is based on the minimization of a quadratic criterion for a linear dynamic system. This control provides the expression of the optimal forces according to the state variables of the system. Knowing the performance criterion and the equations of the system allow us to identify the optimal forces to be applied for better road behaviour, regardless of the constraints related to the state of the road and the stochastic nature of the disturbances.

The variables to be minimized in our model are of two types such as:

Accelerations of the unsprung masses ($\ddot{z}_{rv}$ et $\ddot{z}_{rr}$), which would isolate the vehicle cockpit from irregularities of the road, thus making it possible to improve passenger comfort and vertical displacements of the suspensions (Z_{sv}-Z_{rv}) and (Z_{sr}-Z_{rr}). To optimize the energy contribution necessary for the control forces, we have added the expressions J_5 and J_6 in the criterion of performance defined below.

We define the criterion of performance as follows (Ori *et al*., 2007):

- **J_1, J_2:** Being the quadratic evaluation of front and rear accelerations respectively weighting the constants q_1 and q_2

$$J_1 = \lim_{T\to\infty} \frac{1}{T} \; E \; [\int_0^T q_1 (\ddot{z}_{rv})^2 \; dt] \tag{3}$$

$$J_2 = \lim_{T\to\infty} \frac{1}{T} \; E \; [\int_0^T q_2 (\ddot{z}_{rr})^2 \; dt] \tag{4}$$

- **J_3, J_4:** Averages of squares of relative displacements between the rigid body and the axle, balanced respectively front and rear by the constants q_3 and q_4

$$J_3 = \lim_{T\to\infty} \frac{1}{T} \; E \; [\int_0^T q_3 (z_{sv} - z_{rv})^2 \; dt] \tag{5}$$

$$J_4 = \lim_{T\to\infty} \frac{1}{T} \; E \; [\int_0^T q_4 (z_{sr} - z_{rr})^2 \; dt] \tag{6}$$

- **J_5, J_6:** These terms are useful for limiting the control forces $u_v(t)$, $u_r(t)$ and thus, controlling the energy strain of the suspension

$$J_5 = \lim_{T\to\infty} \frac{1}{T} \; E \; [\int_0^T \rho_v (u_v)^2 \; dt] \tag{7}$$

$$J_6 = \lim_{T\to\infty} \frac{1}{T} \; E \; [\int_0^T \rho_r (u_r)^2 \; dt] \tag{8}$$

The coefficients ρ_v, ρ_r and q_i (i = 1,....,4) are numerical constants whose values give predominance one to or the other of the performances to be achieved. The expression of the criterion will thus be:

$$J = \sum_{i=1}^{6} J_i$$

The square averages of these accelerations reveal a coupling and the index of performance can then be put in the form:

$$J = \lim_{T\to\infty} \frac{1}{T} \ E\left[\int_0^T (U^T R \ U + X^T Q \ X + 2 \,.\, X^T N \, U) \, dt \right] \tag{9}$$

with

$$Q = \begin{bmatrix} q_3 + q_1 . a_1^2 & a_1 . a_2 . q_1 & a_1 . a_3 . q_1 & a_1 . a_4 . q_1 & 0 & 0 & 0 & 0 \\ a_1 . a_2 . q_1 & a_2^2 . q_1 & a_2 . a_3 . q_1 & a_2 . a_4 . q_1 & 0 & 0 & 0 & 0 \\ a_1 . a_3 . q_1 & a_2 . a_3 . q_1 & a_3^2 . q_1 & a_3 . a_4 . q_1 & 0 & 0 & 0 & 0 \\ a_1 . a_4 . q_1 & a_2 . a_4 . q_1 & a_3 . a_4 . q_1 & a_4^2 . q_1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & q_4 + q_2 . a_5^2 & a_5 . a_6 . q_2 & a_5 . a_7 . q_2 & a_5 . a_8 . q_2 \\ 0 & 0 & 0 & 0 & a_5 . a_6 . q_2 & a_6^2 . q_2 & a_6 . a_7 . q_2 & a_6 . a_8 . q_2 \\ 0 & 0 & 0 & 0 & a_5 . a_7 . q_2 & a_6 . a_7 . q_2 & a_7^2 . q_2 & a_7 . a_8 . q_2 \\ 0 & 0 & 0 & 0 & a_5 . a_8 . q_2 & a_6 . a_8 . q_2 & a_7 . a_8 . q_2 & a_8^2 . q_2 \end{bmatrix}$$

$$R = \begin{pmatrix} \rho_v + q_1 . a_{10}^2 & 0 \\ 0 & \rho_r + q_2 . a_9^2 \end{pmatrix} \quad N = \begin{pmatrix} a_1 . a_{10} . q_1 & 0 \\ a_1 . a_{10} . q_1 & 0 \\ a_1 a_{10} . q_1 & 0 \\ a_1 . a_{10} . q_1 & 0 \\ 0 & a_5 . a_9 . q_2 \\ 0 & a_5 . a_9 . q_2 \\ 0 & a_5 . a_9 . q_2 \\ 0 & a_5 . a_9 . q_2 \end{pmatrix}$$

$$a_1 = -k_{sv} / M_{nsv};\ a_2 = -b_{sv} / M_{nsv};\ a_3 = -k_{tv} / M_{nsv};\ a_4 = (b_{sv} - b_{tv}) / M_{nsv};\ a_{10} = 1 / M_{nsv};$$
$$a_5 = -k_{sr} / M_{nsr};\ a_6 = -b_{sr} / M_{nsr};\ a_7 = -k_{tr} / M_{nsr};\ a_8 = (b_{sr} - b_{tr}) / M_{nsr};\ a_9 = 1 / M_{nsr}$$

The values used in the calculation of the criterion are indicated below (Alleyne, 1997):

$M_S = 730.0$ kg	$I_P = 1230.0$ kg $-$ m^2	$r = 0.3$ m
$J_V = 1.4$ kg $-$ m^2	$J_r = 1.0$ kg $-$ m^2	$k_{SV} = 19960.0$ N/m
$k_{Sr} = 17500.0$ N/m	$b_{SV} = 1050.0$ N$-$s/m	$b_{Sr} = 900.0$ N$-$s/m
$l_V = 1.011$ m	$l_r = 1.803$ m	$h = 0.508$ m
Mnsv = 40 kg	Mnsr = 35 kg	$k_{tV} = 17500$ N/m
$k_{tr} = 17500$ N/m	$b_{tV} = 1500$ N/m/s	$b_{tr} = 1500$ N/m/s

The determination of the elements $u_v(t)$ and $u_r(t)$ and the law of control consists of finding the matrix which is the solution of the Riccati equation below:

$$P(t) . A_1 + A_1^T . P(t) + Q - P(t) . B_1 . R^{-1} . B_1^T . P(t) = 0 \tag{10}$$

where P, Q and R are defined, symmetrical and positive matrices.

The command which minimizes this criterion of performance is:

$$U(z,t) = G(t).Z(t) \quad \text{with} \quad G(t) = -R^{-1}.B_1^T.P(t) \tag{11}$$

elements $u_v(t)$ and $u_r(t)$ are obtained writing:

$$\begin{pmatrix} u_v \\ u_r \end{pmatrix} = G.Z \tag{12}$$

Generally, the development of a suspension is a compromise between the minimization of two variables, (acceleration and vertical displacement) but the minimization of vertical displacement does not appear on the same level. So, the choice of the ponderation coefficients and thus of the optimal law determines the control performance. As there is no suitable criterion for determining the parameters, their adjustment is thus made in a dichotomic way.

For the law of control, we chose ponderation coefficients with the following values:

$$\rho_v = 1.75\times10^{-9},\ \rho_r = 1.75\times10^{-9},\ q_1 = 10^{-8},\ q_2 = 10^{-8},\ q_3 = 0.9 \text{ and } q_4 = 2.1$$

We then examine the shape of the curves to conclude on the effectiveness of the law of control.

Appendix B:

```
clear

ksv=19960; ksr=17500; ktv=175500; ktr=175500;

bsv=1050; bsr=900; btv=1500; btr=1500;

ms=730;

mus=40; mur=35;

lf=1.011; lr=1.803; Ip=1230; rho1=1.75*10^(-9); rho2=1.75*10^(-9);

q3=0.9; q4=2.1; q1=10^(-8); q2=10^(-8);

a1=-ksv/mus; a2=-bsv/mus; a3=-ktv/mus; a4=(bsv-btv)/mus; a10=1/mus;
a5=-ksr/mur; a6=-bsr/mur; a7=-ktr/mur; a8=(bsr-btr)/mur; a9=1/mur;

alfa1 = 1./ms + (lf^2)./Ip;
alfa2 = 1./ms - (lf*lr)./Ip;
alfa3 = 1./ms + (lr^2)./Ip;

A=[0 1 0 -1 0 0 0 0;
 -alfa1*ksv -alfa1*bsv 0 alfa1*bsv -alfa2*ksr -alfa2*bsr 0 alfa2*bsr;
 0 0 0 1 0 0 0 0;
```

```
 -ksv/mus -bsv/mus -ktv/mus (bsv-btv)/mus 0 0 0 0;
 0 0 0 0 0 1 0 -1;
 -alfa2*ksv -alfa2*bsv 0 alfa2*bsv -alfa3*ksr -alfa3*bsr 0 alfa3*bsr;
 0 0 0 0 0 0 0 1;
 0 0 0 0 -ksr/mur -bsr/mur -ktr/mur (bsr-btr)/mur];

Q=[q3+q1*(a1)^2 a1*a2*q1 a1*a3*q1 a1*a4*q1 0 0 0 0;
 a1*a2*q1 q1*(a2)^2 a2*a3*q1 a2*a4*q1 0 0 0 0;
 a1*a3*q1 a2*a3*q1 q1*(a3)^2 a3*a4*q1 0 0 0 0;
 a1*a4*q1 a2*a4*q1 a3*a4*q1 q1*(a4)^2 0 0 0 0;
 0 0 0 0 q4+q2*(a5)^2 a5*a6*q2 a5*a7*q2 a5*a8*q2;
 0 0 0 0 a5*a6*q2 q2*(a6)^2 a6*a7*q2 a6*a8*q2;
 0 0 0 0 a5*a7*q2 a6*a7*q2 q2*(a7)^2 a7*a8*q2;
 0 0 0 0 a5*a8*q2 a6*a8*q2 a7*a8*q2 q2*(a8)^2];

B=[0 0;
 alfa1 alfa2;
 0 0;
 1/mus 0;
 0 0;
 alfa2 alfa3;
 0 0;
 0 1/mur];

R=[rho1+q1*(a10)^2 0; 0 rho2+q2*(a9)^2];

N=[a1*a10*q1 0;
 a2*a10*q1 0;
 a3*a10*q1 0;
 a4*a10*q1 0;
 0 a5*a9*q2;
 0 a6*a9*q2;
 0 a7*a9*q2;
 0 a8*a9*q2];

[K, S, E] = lqr (A, B, Q, R, N);
disp ('the matrix which is the solution of the Riccati equation is :')
K
```

NOMENCLATURE

x : Vehicle center of gravity forward position

z : Vehicle center of gravity vertical position

I_p : Vehicle pitch inertia about center of gravity

θ : Vehicle pitch angle about center of gravity

h : Height of center of gravity from road

z_t : Road displacement
z_r : Axle displacement
z_s : Sprung mass displacement
k : Suspension spring constant
b : Suspension damping constant
u : Active force element
l_f : Distance from center of gravity to vehicle front wheel
l_r : Distance from center of gravity to vehicle rear wheel
$v = \dot{x}$: Vehicle longitudinal velocity
w : Wheel angular velocity
$\dot{w}$: Wheel angular acceleration
T_b : Brake torque from disk
J : Wheel inertia
r : Effective tire radius

REFERENCES

Alleyne, A., 1997. Improved vehicle performance using combined suspension and braking forces. Vehicle Syst. Dyn., 27: 235-265.

Barzamini, R., A.R. Yazdizadeh, H.A. Talebi and H. Eliasi, 2009. Adaptive control of a double-electromagnet suspension system. J. Applied Sci., 9: 1201-1214.

Baslamisli, S.C., I.E. Kose and G. Anlas, 2009. Gain-scheduled integred active steering and differential control for vehicle handling improvement. Vehicle Syst. Dyn., 47: 99-119.

Frendo, F., A. Sisi, M. Guiggiani and S. Di Piazza, 2006. Analysis of motorcycle models for the evaluation of the handling performances. Vehicle Syst. Dyn., 44: 181-191.

Ori, T.R., 2001. Suspensions actives et comportement dynamique des vehicules lors du freinage: Active suspensions and vehicles dynamic behaviour during braking phase. Ph.D. Thesis, Ecole Centrale de Lyon.

Ori, T.R., M.N. Ichchou, P. Gbaha and L. Jezequel, 2007. Improvement of handling by means of active suspension control. Eng. Trans., 55: 155-180.

Pacejka, H.B., 2006. Tyre and Vehicle Dynamics. 2nd Edn., Elsevier Science Publication, New York, ISBN-13: 980-0-7506-6918-4, pp: 156-215.

Rashid, M.M., M.A. Hussain and N. Abd. Rahim, 2006. Application of magneto-rheological damper for car suspension control. J. Applied Sci., 6: 933-938.

Wu, J.D. and R.J.R.J. Chen, 2004. Application of an active controller for reducing small-amplitude vertical vibration in vehicle seat. J. Sound Vibration, 274: 939-951.

Zhang, Y. and A. Alleyne, 2005. A practical and effective approach to active suspension control. Vehicle Syst. Dyn., 43: 305-330.