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STATISTICAL ENERGY ANALYSIS : FROM THE UNDERSTANDING OF ITS ASSUMPTIONS TO THE PRACTICAL USE.

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ABSTRACT

Statistical Energy Analysis (SEA) is in the field of energy methods. It aims at describing the energy transfers between complex subsystems. This method is regularly presented as a solution to bypass the problems which can arise in the high frequency range when applying deterministic methods such as FEM (cost of computation due to the high number of degree of freedom or the high number of modes, unicity of the computation). But SEA has numerous assumptions which are sometimes forgotten or misunderstood. Indeed, the industrial applications of the method have often been disappointing and the lack of strictly defined rules brings SEA into disrepute. This paper recalls SEA assumptions distinguishing the modal and the wave approach. The goal is to study the possible equivalence between the hypotheses and their influence on the quality of the results. Simple examples are taken (coupled oscillators, couples plates) to illustrate the observations. Some guidelines are extracted and they are applied on a plate network. The modal energy equipartition assumption is shown equivalent to the rain-on-the-roof assumption. Moreover, the latter implies diffuse field assumptions. The importance of the weak coupling assumption is also broached. It is observed that indirect coupling loss factors come up for strong coupling regimes making the coupling power proportionality relationship unusable.

1 INTRODUCTION

The statistical energy analysis (SEA) is a method introduced in the 1960s intended to estimate the vibroacoustic response of complex structures in the high frequency range by a statistical approach. Richard Lyon's book [1] is a benchmark in this field, but many writers have contributed to the development of this method. Purely theoretical developments of traditional SEA have occurred between the early 60s and the 80s. After, the literature is mainly concerned with reviews, experiences and some extensions about SEA.

The main results given by SEA is the coupling power proportionality relationship (CPP). It states that the power transmitted between two subsystems is proportional to the difference of their modal energies [2]. From a thermodynamic point of view the CPP can be seen as a relaxation phenomenon of nonequilibrium systems [3]. Indeed, it is a linear relationship between the flow and the thermodynamic forces, whose definition is a difference of energies.

SEA (or the CPP relationship), to be applied properly, requires a number of assumptions often misunderstood or unknown. There are many studies using SEA that led to disappointments. It has motivated studies on the required assumptions and have also divided the opinions of the scientific community on their status. A study of the equivalence or the effective need of SEA assumptions could be beneficial and it is precisely the subject of this paper. For that four parts follow this introduction. First a brief review of SEA assumptions is done distinguishing the two approaches of SEA [4]: the modal and wave approaches. Second, the results of a study on the equivalence between rain-on-the-roof excitation, diffuse field and modal energy equipartition is presented. Then, results from another study about the weak coupling hypothesis is shown. The observations of both studies are finally used on an example of three coupled plates.

2 BRIEF REVIEW OF SEA ASSUMPTIONS

The number of SEA assumptions is linked to the plurality of the demonstrations to prove the coupling power proportionality. To understand their origin, the two approaches of SEA (modal and wave) must be recalled. A detailed review with the mathematical demonstrations is given by Le Bot [5] and a summary version is given in chapter two of [6].

2.1 Modal approach

The modal approach considers a mode as a mechanical oscillator with a mass, an internal damping and a stiffness. In this way, the energy exchange between two modes is developed on the model of two coupled oscillators. The more the system evolves, the more the demonstration model becomes complex (exchange between three modes, between N modes, between several subsystems, between continuous systems). For each demonstration, the CPP can be recovered through the consideration of several assumptions. The hypotheses are the following:

1. *The coupling between two modes is conservative*. With this assumption no power is created or dissipated in the coupling.

2. *Modes are excited by random, stationary and uncorrelated external forces*. From a mathematical point of view, assuming this kind of excitation allows to consider the expectations of each term of the power balance equation instead of their intantaneous expressions. Moreover, the properties for two stationary random processes can then be used. From a physical point of view, this excitation enables to have the same power injected in each mode.

3. *The external forces are white noises*. It means that the power spectral densities of the excitations are flat over an infinite frequency band.

4. *The coupling is weak*. We may note that this assumption comes up when the model consists in three oscillators or more. Indeed, for two coupled modes assumptions 1, 2 and 3 are enough to prove the CPP. No consideration on the coupling is needed. In this way the demonstration model with two oscillator is a particular case.

5. *The power spectral densities of the excitations are identical within each subsystem.* This hypothesis is necessary for the demonstration model of sets of oscillators. Modes within a subsystem are uncoupled but each mode of a subsystem i can be coupled to any other mode of a subsystem j. It complements the assumptions 2 and 3. In summary, the excitations are stationary random, uncorrelated white noise and their spectral power densities are the same in all subsystems. This type of excitation is also called *rain-on-the-roof* because the forces are spatially and temporally uncorrelated and of equal intensity in analogy with raindrops falling on a roof.

6. *The number of mode is large in each subsystem.* The CPP between two subsystems (i.e sets of oscillators) is similar to the CPP for $N > 2$ coupled oscillators with a summation over the number of modes within each subsystem. The main drawback is that all of the oscillators parameters must be known. To avoid this issue one can make a probabilistic estimation of the sums by replacing them with integrals. This additional assumption allows to minimize the error in this latter operation and makes the transition from a deterministic problem into a probabilistic problem.

7. *The probability density function are unchanging over the frequency band $\Delta\omega$.* Previously the resonators were deterministic and deterministic problem solving required a total knowledge of all oscillators parameters. In practice the subsystems contain a large number of modes; the number of oscillators and of parameters is then very high. Conversely, the problem in its probabilistic form requires knowing only the statistical properties of an oscillator. They are precisely specified by this assumption.

8. *The damping is small.* This is the latest hypothesis required for the demonstration of the CPP for coupled sets of oscillators.

9. *Equipartition of modal energy*. Another method is used to prove the CPP without a formal calculation of energies. In this approach assumption 5 is replaced with this assumption. It permits to write the vibrational energy of an oscillator as the vibrational energy of the whole subsystem divided by the total number of oscillators. It is important to note that this assumption of equipartition of modal energy is a behavioral assumption and consequently difficult to verify a priori.

2.2 Wave approach

So far the approach used was the modal approach. It helped to identify the source of nine hypotheses of SEA. The wave approach assumes that the vibrational field is decomposed into a superposition of traveling plane waves sufficiently disordered to address the problem in a statistical manner. The required assumption to demonstrate the coupling power proportionality are:

10. *High frequencies.* This hypothesis allows the approximation of geometrical vibroacoustics and use ray method.

11. *Diffuse field.* There are several definitions of a diffuse field. From the ray method point of view a diffuse field is a sum of plane waves of amplitudes and random phases, from all directions in an equiprobable way. At one point the energy is calculated by integrating the contributions from the entire space.

12. *Weak coupling*. The subsystems adjacent border is small relative to other frontier. Giving these three assumptions one is able to recover the CPP relationship.

To have fast and reliable results using SEA, an operator will have some difficulties to check all the prescribed assumptions. While he/she may control the excitation, it will not be easy for him/her to know the status of subsystems. On the other hand, the two approaches lead to the same result: the CPP. We may wonder about the possible equivalence between the assumptions of each approach. In this context, a study of the inclusions, the equivalence and the need of SEA assumptions could be beneficial. This is precisely the subject of the following parts.

3 EQUIVALENCE BETWEEN RAIN-ON-THE-ROOF, DIFFUSE FIELD AND MODAL ENERGY EQUIPARTITION ASSUMPTIONS

A study about the equivalence between rain-on-the-roof excitation, diffuse field and modal energy equipartition was undertaken. First, the example of a simply supported plate excited by a random point force has been processed to study the conditions of diffuseness. A diffuse field criterion based on the standard deviation of the local energy at several random point on the plate has been defined. This criterion was drawn on the wavenumber/damping plan (cf Figure 1).

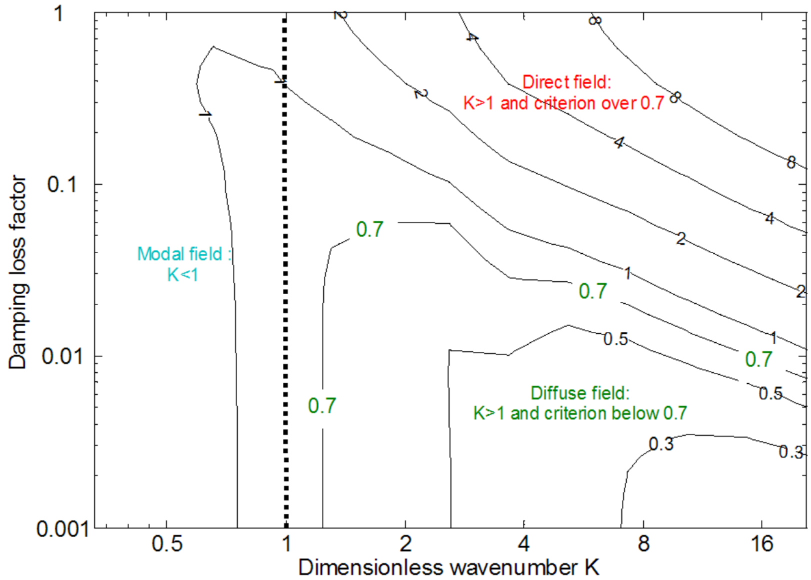

Figure 1: Diffuse field criterion on the wavenumber/damping plan for a general rectangular plate excited by a single point force. The lines represent the diffuse field criterion value while the damping and the wavenumber vary.

The contour line 0.7 defines the area of quasi-diffuse field. It corresponds to small damping plus high frequencies conditions. For lower frequencies, where few modes are resonant, the field has a modal behavior, whereas at high frequencies and for strong damping the field is dominated by a direct field emanating from the source. Consequently, in a general rectangular plate submitted to a single point force, diffuse field is possible in the high frequency range and for small damping.

When the plate is excited by a rain-on-the-roof excitation (i.e a infinite number of uncorrelated, white noise, random point forces) we observed that the diffuse field domain becomes larger. Indeed, we noted that the diffuse field criterion dramatically decreases when the number of excitations increases. It means that, in a subsystem, if a field is not naturally diffuse with a single point force (modal field or direct field behavior) it becomes diffuse when a rain-on-the-roof is used. Regarding the assumptions one can conclude that the hypothesis of a rain-on-the-roof excitation implies the hypothesis of diffuse field:

Assumptions (2)+(3)+(5) $\Rightarrow$ Assumption (11).

The same example is taken to broach the modal energy equipartition assumption. As for the diffuse field an equipartition criterion has been defined. It is the standard deviation divided by the mean value of modal energy. The results show that equipartition is never reached with a single point force even for small damping and high frequencies. The modal energies follow a Weibull distribution which shows that it is not evenly distributed. Nevertheless, equipartition is possible when a rain-on-the-roof is used and when the damping is structural (also denoted half power bandwidth). In that case the modal energies are group around a mean value. The modes of subsystem do not interact but they all receive the same injected power (provided by the rain-on-the-roof) and dissipate at the same rates (half power bandwidth dissipation). For these reasons, if one considers implicitly structural damping, the rain-on-the-roof assumptions is strictly equivalent to modal energy equipartition:

Assumptions (2)+(3)+(5) ⇔ Assumption (9).

Finally the study has allowed to decide on the equivalence between rain-on-the-roof, diffuse field and modal energy equipartition (cf Figure 2).

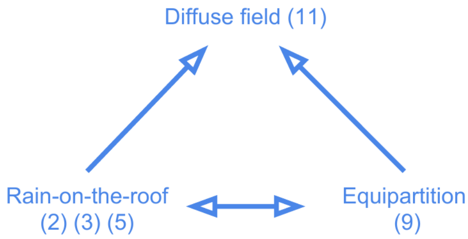

Figure 2: Equivalence and implications between rain-on-the-roof, diffuse field and modal energy equipartition assumption in SEA.

The reader is encouraged to read the paper [7] for more details on this study.

4 ON THE WEAK COUPLING REQUIREMENT

One focused on the weak coupling assumption as it appears in both approaches of SEA. The aim here was to verify some results of the literature and to study the effects of a strong coupling regime of the CPP.

First one took two and three oscillators of stiffness k coupled by springs of stiffness K [1] to verify an important result: the need to check the weak coupling assumption for a energy exchange between the three modes (or more). To do so, the traditional coupling loss factor β given by the SEA literature has been compared with a reference factor while the coupling strength varies. The reference coupling loss factor is the ratio between the exact power transmitted and the difference between the exact energies of each oscillator. The conclusion is final: a perfect agreement between SEA and reference calculation for two coupled oscillators whatever the coupling strength ratio K/k is showing that this assumption is not necessary to apply SEA properly (cf Figure 3).

At the opposite, the case of three coupled oscillators shows that a separation line between weak and strong coupling can be drawn (cf Figure 4 (a) and (b)). For weak coupling the SEA and reference match. But, there is a large difference between $\beta_{ij,SEA}$ and $\beta_{ij,REF}$ when the coupling ratio K/k is above a threshold value ($K/k = 0.1$ in both figures). In this way, weak coupling is a unavoidable to apply SEA.

The case of three coupled plates has also been examined. It has been observed that in the strong coupling regime the CPP fails giving also rise to indirect coupling loss factors. These results led to reference [8].

[1] The oscillators are coupled linearly : one spring is use for the case of two coupled oscillators and two for the case of three coupled oscillators.

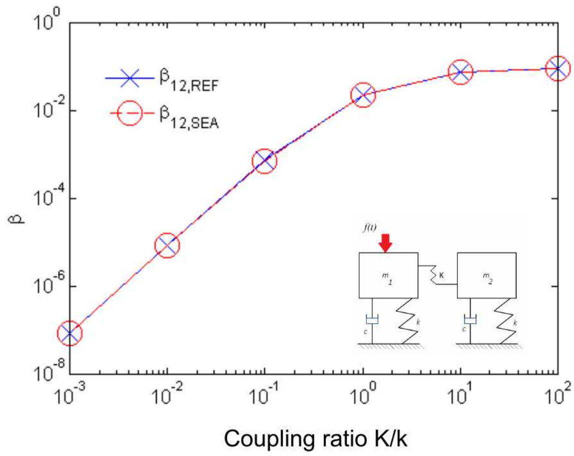

Figure 3: Evolution of the coupling power coefficient of SEA $\beta_{12,SEA}$ and the reference $\beta_{12,REF}$ between two oscillators versus the stiffness ratio K/k.

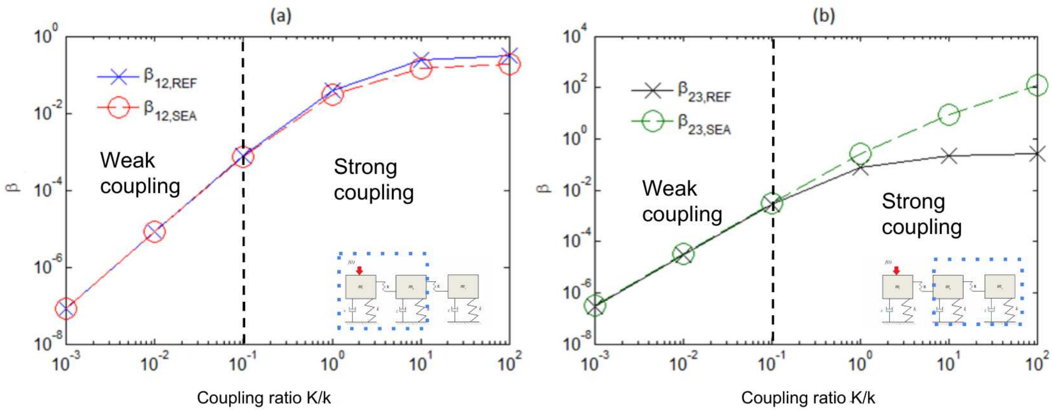

Figure 4: Evolution of the coupling power coefficient of SEA $\beta_{ij,SEA}$ and the reference $\beta_{ij,REF}$ between three oscillators versus the stiffness ratio K/k where (i,j) = (1,2) in (a) and (i,j) = (2,3) in (b).

5 APPLICATION ON A PLATE NETWORK

In this section each hypothesis (diffuse field, rain-on-the-roof, equipartition, weak coupling) is tested on a network of three coupled plates. Several configurations are considered, for each the SEA calculation is compared to an analytical calculation (reference). Let us firstly present these two calculations.

5.1 SEA calculation

The plates are referenced A, B and C. Their length and width are denoted L_x^i and L_y^i. The coupling stiffness between plates i and j (where $i = A, B$ and $j = B, C$) are K_{ij} (cf Figure 5). If P_i is the power injected in plate i and E^i its mean vibrational energy, then the SEA system when only plate A is excited is

$$\frac{1}{\omega_c}\begin{pmatrix} P_A \\ 0 \\ 0 \end{pmatrix} = \begin{bmatrix} \eta_A + \eta_{AB} & -\eta_{BA} & 0 \\ -\eta_{AB} & \eta_B + \eta_{BA} + \eta_{BC} & -\eta_{CB} \\ 0 & -\eta_{BC} & \eta_C + \eta_{CB} \end{bmatrix} \begin{bmatrix} E_A \\ E_B \\ E_C \end{bmatrix} \tag{1}$$

where η_i and η_{ij} are the damping and the coupling loss factors. Besides, in [9] Mace and Li give the coupling loss factor between two coupled plates by a spring. With the present notation such an

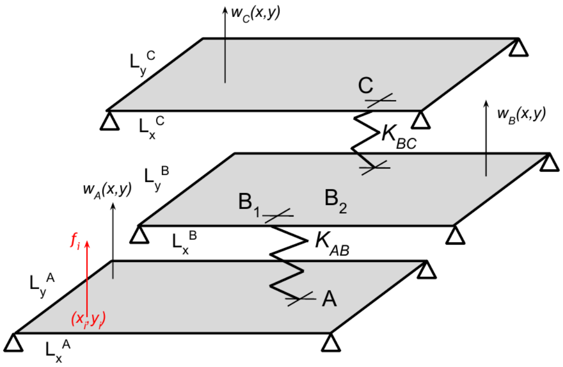

Figure 5. Three simply supported plates A, B and C coupled by elastic couplings K_{AB} and K_{BC}.

expression is, for plate A and B,

$$n_A \eta_{AB} = n_B \eta_{BA} = \frac{K_{AB}^2}{32\pi\omega^3} \frac{1}{\sqrt{\rho_A h_A D_A}\sqrt{\rho_B h_B D_B}}, \tag{2}$$

where ω is the frequency band, ρ_i, h_i and D_i the density, the thickness and the bending stiffness. The asymptotic expression of modal density n_i in plate i is

$$n_i = \frac{L_x^i L_y^i}{4\pi} \sqrt{\frac{\rho_i h_i}{D_i}} \tag{3}$$

The development of the second and third line of (1) leads to two energy ratios:

$$\left(\frac{E_B}{E_A}\right)_{SEA} = \frac{\eta_{AB}}{[(\eta_B + \eta_{BA} + \eta_{BC}) - \eta_{CB}\frac{\eta_{BC}}{\eta_C + \eta_{CB}}]} \tag{4}$$

and

$$\left(\frac{E_C}{E_A}\right)_{SEA} = \frac{\eta_{BC}}{\eta_C + \eta_{CB}} \times \frac{\eta_{AB}}{[(\eta_B + \eta_{BA} + \eta_{BC}) - \eta_{CB}\frac{\eta_{BC}}{\eta_C + \eta_{CB}}]} \tag{5}$$

Finally, the energy ratios predicted by SEA can be obtained by the computation of the coupling loss factors and the modal densities (the damping loss factors are assumed to be known).

5.2 Reference calculation

If plate A is excited by a rain-on-the-roof excitation composed of N random point forces, the force field f has the following expression

$$f(x, y, t) = \sum_{k=1}^{N} F_k(t)\delta(x - x_k)\delta(y - y_k) \tag{6}$$

where F_k are random forces having the same power spectral density S_A constant in the frequency band $[\omega_{\min}, \omega_{\max}]$. The vibrational energy is taken as twice the kinetic energy $E_i = \int \rho_i h_i \langle \dot{w}_i^2 \rangle dxdy$ where w_i denotes the deflection of plate i and the integral is performed over the plate surface. The vibrational energies of each plate can be computed by using the frequency response functions of the coupled plates. The deflection w_i of plate i at position x, y when plate A

is excited by a harmonic point force at position x_k, y_k is noted $H_{i,A}(x, y, x_k, y_k; \omega)$ where ω is the circular frequency. The vibrational energy in plate i are admitted

$$E_i = \frac{S_A}{\pi} \sum_{k=1}^{N} \int_0^{L_x^i} \int_0^{L_y^i} \int_{\omega_{\min}}^{\omega_{\max}} \rho_i h_i \omega^2 |H_{i,A}(x, y, x_k, y_k; \omega)|^2 d\omega dy dx \quad (7)$$

We may note that the computation of the each vibrational energy is reduced to the computation of the frequency response functions of the coupled plates H_{ij}. The reader may refer to [7] for a detailed computation of the energies of two coupled plates and to [8] or [6] chapter 7 for three coupled plates. The energy ratios $(E_C/E_A)_{REF}$ and $(E_B/E_A)_{REF}$ are consequently obtained by computing the relevant energies from equation (7). SEA and analytical calculation can then be compared with the difference

$$\Delta_{SEA-REF}^{A\to B} = \left| 10\log\left(\frac{E_B}{E_A}\right)_{SEA} - 10\log\left(\frac{E_B}{E_A}\right)_{REF} \right| \quad (8)$$

Of course the difference between plate A and C is similar and is denoted $\Delta_{SEA-REF}^{A\to C}$.

5.3 Numerical tests

With equation (8) one is now able to compare the SEA and the calculation reference. The simulation parameters are given in the Table 1. Six configurations are tested and for all the frequency

Type	Symbol	Value	Unity
Plate A	$L_x^A \times L_y^A$	1.44×1.2	m^2
Plate B	$L_x^B \times L_y^B$	1.39×1.1	m^2
Plate C	$L_x^C \times L_y^C$	1.42×1.3	m^2
Thickness	h_A=h_B=h_C	2	mm
Density	ρ	7800	kg/m^3
Young modulus	E	$2.1E11$	N/m^2
Poisson coefficient	ν	0.3	$-$
Attachment point A	x_A, y_A	$0.72, 0.61$	m
Attachment point B1	x_{B1}, y_{B1}	$0.35, 0.91$	m
Attachment point B2	x_{B2}, y_{B2}	$1.09, 0.31$	m
Attachment point C	x_C, y_C	$0.35, 0.30$	m

Table 1. General parameters of the three coupled plates.

band $\Delta\omega$ is an octave centred on 2 kHz (corresponding to $\kappa = 10.3$ in Figure 1). The results are shown with Table 2. When a hypothesis is actually true, the box is marked ”Yes” (or ”weak” for the hypothesis of coupling). If not it is marked ”No” (or ”strong”).

Clarification of the rules:

- When the diffuse field box of a plate i is marked ”Yes”, this means that the simulation parameters were set so that the plate is in the area of diffuse field ($\kappa > 1$ and the criterion is below 0.7). This means that the plate i has a damping coefficient of $\eta_i = 0.02$.
- The boxes ”equiparition” and ”rain-on-the-roof” are linked since the two assumptions are equivalent. When the checkbox ”rain-on-the-roof” is marked ”Yes” it means that the plate is subjected to a field of 100 random stationary, uncorrelated white noise excitations. Automatically the equiparition box is marked ”Yes”. Otherwise the two boxes are filled ”No”.

- Finally, the weak coupling hypothesis is verified when the stiffness of coupling between the two plates is less than the threshold value. Like the example of the three coupled oscillators (Figure 3), this value separates the two coupling regimes and depends on the properties of each plate. In our example it corresponds to $K = 10^6$ N/m (cf chapter 7 of [6]). Thus, when the weak coupling assumption is marked "Yes" the coupling stiffness K_{AB} or K_{BC} is equal to 10^4 N/m. Otherwise it is equal to 10^7 N/m.

Test Case	Hypotheses	Plate A	Plate B	Plate C	$\Delta^{A\to B}_{SEA-REF}$	$\Delta^{A\to C}_{SEA-REF}$	Conclusion
1	Diffuse field domain (yes/no)	No	Yes	Yes	2.6 dB	2.9 dB	SEA is practicable
	Equipartition of modal energies (y/n)	Yes	No	No			
	Rain-on-the-roof excitation (y/n)	Yes	No	No			
	Coupling (weak/strong)	Weak		Weak			
2	Diffuse field domain (yes/no)	Yes	Yes	Yes	1.5 dB	3.7 dB	SEA is practicable
	Equipartition of modal energies (y/n)	No	No	No			
	Rain-on-the-roof excitation (y/n)	No	No	No			
	Coupling (weak/strong)	Weak		Weak			
3	Diffuse field domain (yes/no)	No	Yes	No	2.0 dB	2.6 dB	SEA is practicable
	Equipartition of modal energies (y/n)	Yes	No	No			
	Rain-on-the-roof excitation (y/n)	Yes	No	No			
	Coupling (weak/strong)	Weak		Weak			
4	Diffuse field domain (yes/no)	No	Yes	Yes	26.2 dB	28.7 dB	SEA is not practicable
	Equipartition of modal energies (y/n)	Yes	No	No			
	Rain-on-the-roof excitation (y/n)	Yes	No	No			
	Coupling (weak/strong)	Strong		Weak			
5	Diffuse field domain (yes/no)	No	Yes	Yes	3.8 dB	25.1 dB	SEA is not practicable
	Equipartition of modal energies (y/n)	Yes	No	No			
	Rain-on-the-roof excitation (y/n)	Yes	No	No			
	Coupling (weak/strong)	Weak		Strong			
6	Diffuse field domain (yes/no)	No	No	No	2.5 dB	12.4 dB	SEA is not practicable
	Equipartition of modal energies (y/n)	Yes	No	No			
	Rain-on-the-roof excitation (y/n)	Yes	No	No			
	Coupling (weak/strong)	Weak		Weak			

Table 2. Test case on the plate network.

At the sight of these tests, it appears that when the weak coupling assumption is not verified it automatically provokes the failure of SEA (test case 4 and 5). SEA can work if the indirectly excited subsystems do not meet the assumption of equipartition of modal energy (or the assumption of rain-on-the-roof). In that case, the diffuse field hypothesis must be verified in all subsystems otherwise SEA fails. In order to respect such an assumption, either the subsystem are within the diffuse field domain; or they must be excited by a rain-on-the-roof to enforce diffuse field (test case 1, 2 and 6). Test case 3 shows that SEA may possibly be applicable if the latter indirectly excited subsystem is not in diffuse field conditions. Nevertheless, this very particular configuration is not recommended.

6 CONCLUSION

Statistical energy analysis in a statistical approach of vibroacoutics which describes complex systems in terms of vibrating and acoustical energies. In the high frequency range, this method constitutes an alternative to bypass the problems which can arise when applying deterministic methods. But its use requires the knowledge and the fulfillment of strong assumptions which restrict its domain of application.

In this paper diffuse field, equipartition of modal energy, weak coupling and the rain-on-the-roof excitation are the took up hypotheses. Their equivalence and their influence on the results quality have been discussed to contribute to the clarification of the necessary assumptions to apply properly SEA. The industrial applications with SEA have often been disappointing because of the violation of some hypotheses.

An first interesting result is that a diffuse field is possible in a system excited by a single point force if it has small damping and in the high frequency range. Secondly, equipartition of modal energy is foreseeable when a rain-on-the-roof excitation (which implies a constant power injected for all modes) and a half power bandwidth damping (constant dissipation for each mode) are gathered. In this way, the rain-on-the-roof hypothesis is equivalent to modal energy equipartition hypothesis. Furthermore, a rain-on-the-roof imposes a diffuse field in the subsystem. Another conclusion is that the rain-on-the-roof hypothesis implies the diffuse field hypothesis. It has an interesting practical implication: imposing a rain-on-the-roof strictly implies the fulfillment of the modal energy equipartition assumption and the diffuse field assumption. This is good news because the two latter assumptions are not easily verified for industrial applications.

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