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Laboratoire de Tribologie et Dynamique des Systèmes

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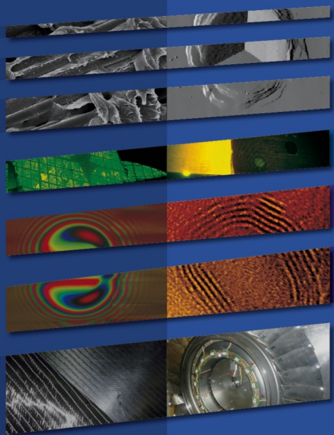

High frequency vibroacoustics: a radiative transfer equation based approach

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InnoWave 2012 – Innovation in wave modelling
3 – 7 september 2012, Nottingham, UK

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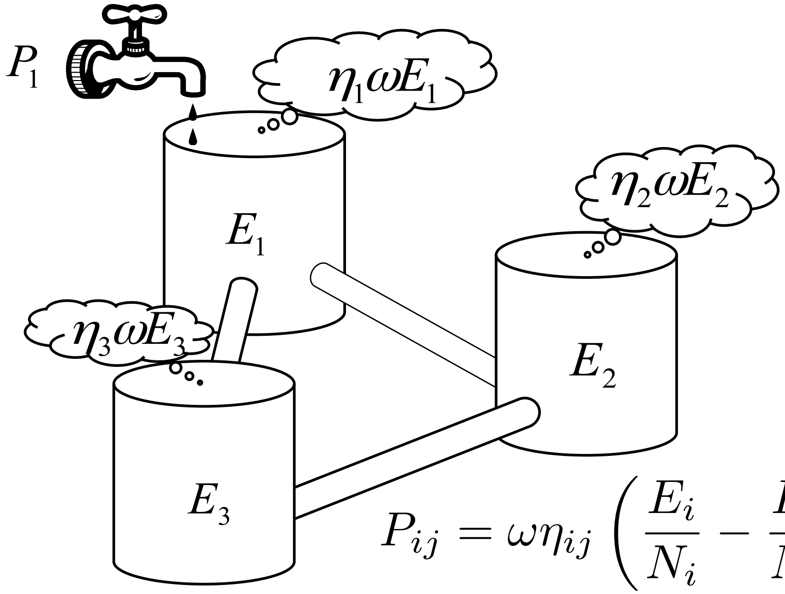
Statistical Energy Analysis



$$P_{ij} = \omega \eta_{ij} \left(\frac{E_i}{N_i} - \frac{E_j}{N_j} \right)$$

Assumptions

- Conservative coupling,
- Light coupling,
- Random forces, white noise
- Large number of modes,
- Large modal overlap,
- Diffuse field

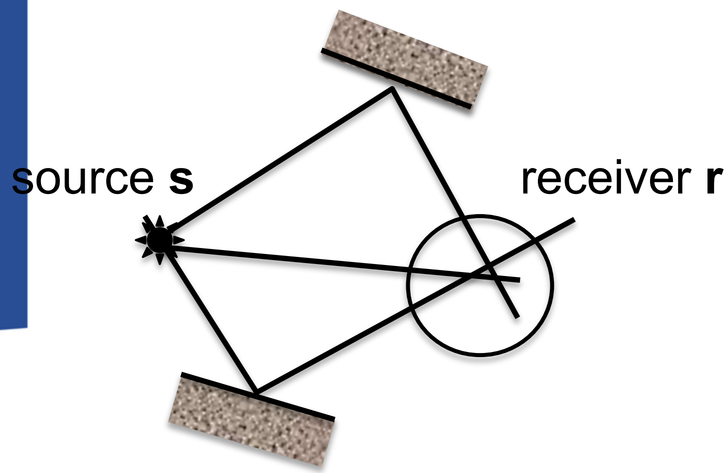
Main features

- Statistical description of systems
- Primary variable = energy,
- Linear superposition

But

- **no locality**

Ray tracing



Assumption

- High frequency,

Two variants:
- rays with phase
- rays without phase (uncorrelated)

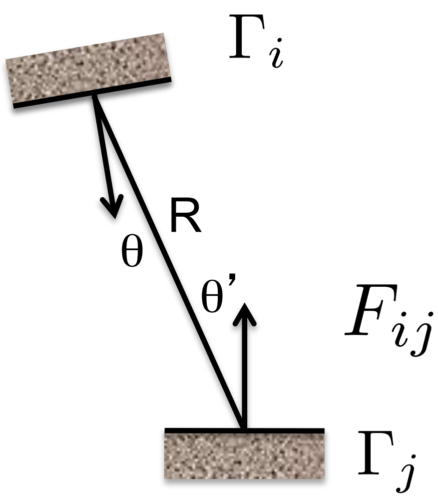
Main features

- Local description
- Linear superposition principle
- Primary variable = field OR energy

But

Requires the determination of all ray paths

View factor



$$F_{ij} = \int_{\Gamma_i} \frac{\cos\theta\cos\theta'}{\pi R^2} d\Gamma$$

Assumption

- High frequency,

Main features

- Local description
- Linear superposition principle
- Primary variable = energy

But

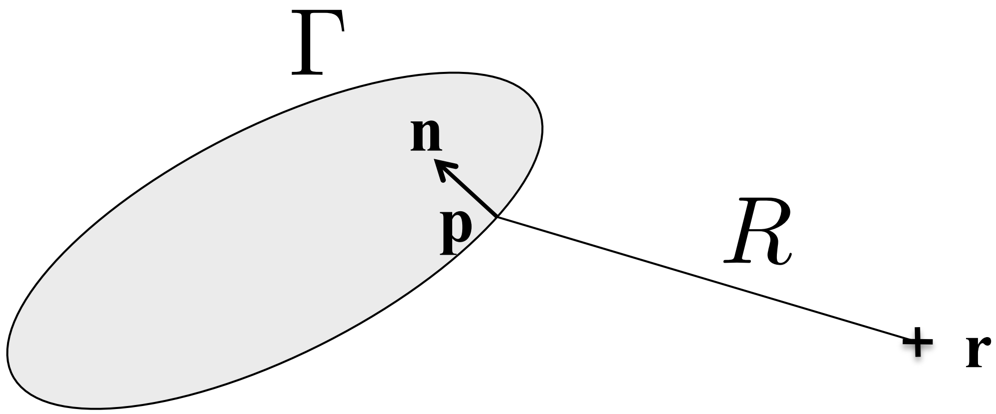
Limited to Lambertian reflection

Objective

High frequency theory of waves in vibroacoustics,

i) Primary variables = power & energy
ii) Local description
iii) Reflection, diffraction, transmission, radiation…
iv) Statistical energy analysis as a special case

Helmholtz-Kirchhoff equation



$$\psi(\mathbf{r}) = \int_\Gamma \underbrace{a(\mathbf{p})g(\mathbf{r},\mathbf{p})}_{\text{single layer}} - \underbrace{b(\mathbf{p})\partial_n g(\mathbf{r},\mathbf{p})}_{\text{double layer}} d\Gamma_{\mathbf{p}}$$

$$a = \partial_n \psi$$
$$b = \psi$$

The reflected field in domain Ω may be constructed by superimposing a single layer and a double layer.

Green's function $$g(\mathbf{p},\mathbf{r}) = \frac{e^{-ikR}}{4\pi R}$$

$$|\psi|^2 = \int_{\Gamma\times\Gamma} aa'gg' + ab'g\partial_n g' + a'bg'\partial_n g + bb'\partial_n g\partial_n g' d\Gamma d\Gamma'$$

Uncorrelation assumption: cross product vanish

$$\int_{\Gamma\times\Gamma} d\Gamma d\Gamma' \sim \int_{\Gamma} d\Gamma$$

Far-field $|g|^2 \propto g\partial_n g \propto |\partial_n g|^2 \propto 1/R^2$

$$|\psi|^2 = \int_{\Gamma} \sigma \times \frac{d\Gamma}{R^2}$$

A vibroacoustic model for high frequency analysis, A. Le Bot, J. Sound Vib. 1998.

Uncoherent energy field

$$\psi(\mathbf{r}) = \int_\Gamma a \times \frac{e^{ikR}}{4\pi R} - b \times \partial_n \frac{e^{ikR}}{4\pi R} d\Gamma$$

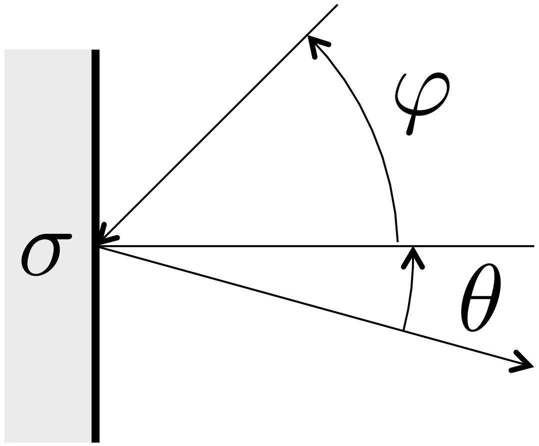
$$\Downarrow$$

$$|\psi|^2 = \int_\Gamma \sigma(\mathbf{p}, \theta) \times \frac{d\Gamma_{\mathbf{p}}}{R^2}$$

The single and double layers of field reduce to a single layer of energy

How to determine the layer σ?

Directivity



Memoryless law $\sigma(\mathbf{p}, \theta) = \sigma(\mathbf{p}) \times f(\theta)$

Lambert's reflection $f(\theta) = \cos\theta$

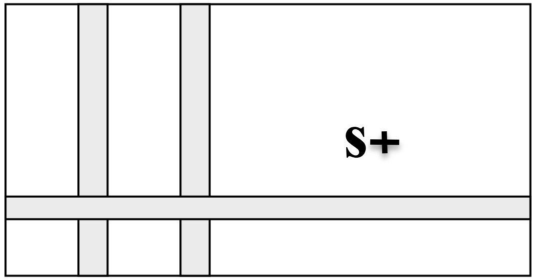
Memory law $\sigma(\mathbf{p}, \varphi, \theta)$

Specular reflection $\sigma(\mathbf{p}, \theta) = \sigma\delta(\varphi - \theta)$

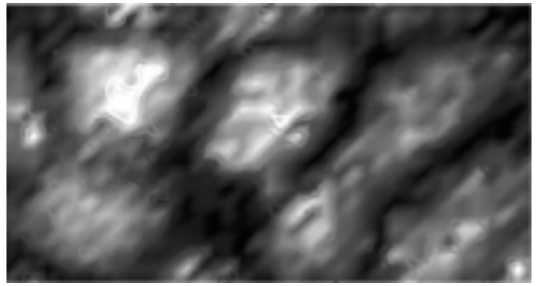
Sabines reverberation time and ergodic audotorium, Joyce, J. Acoust. Soc. Am. 1975.

Sound radiation

Ribbed panel

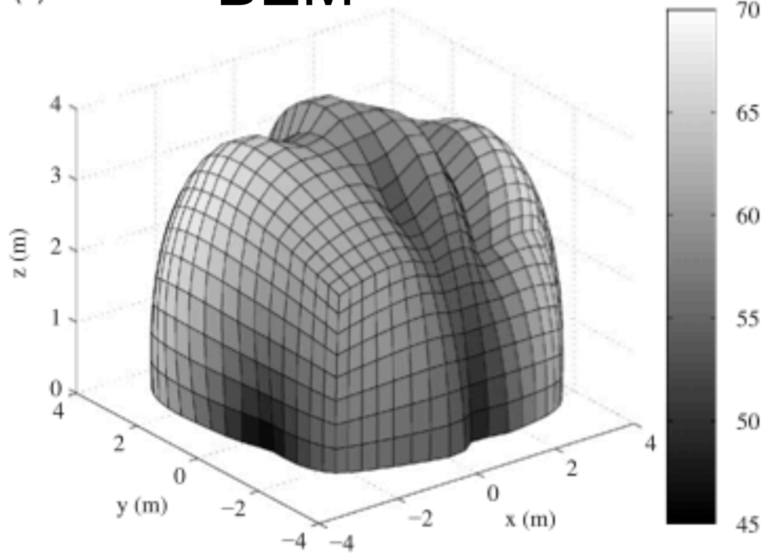


Velocity field

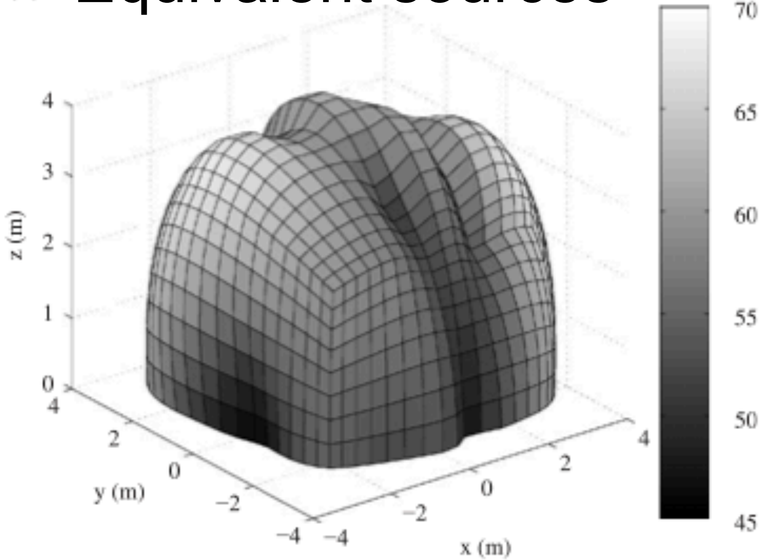


SPL on a sphere of radius 3m – third-octave 2500 Hz

(a) BEM

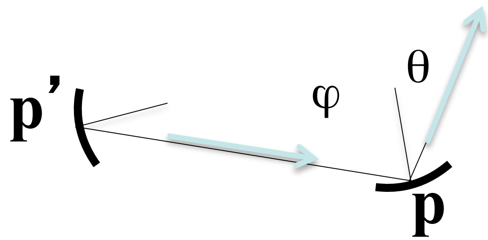


(b) Equivalent sources



A hybrid method for vibroacoustics based on the radiative transfer method, E. Reboul & al., J. Sound Vib. 2007.

Radiosity integral



Assumption

- Lambert's law,
- $\mathcal{R}$ reflection coefficient

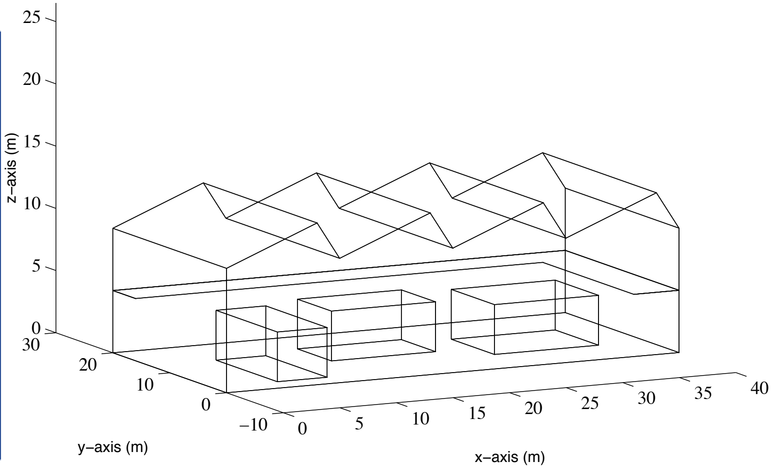
$$\mathcal{P}_{\mathrm{ref}} = \mathcal{R}\mathcal{P}_{\mathrm{inc}}$$

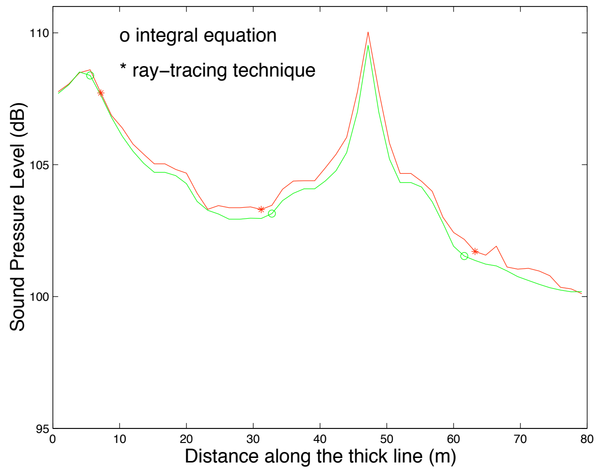
$$\underbrace{\sigma(\mathbf{p},t)}_{\text{Reflected power}} = \mathcal{R}\underbrace{\left(I_0 + \int_\Gamma \sigma(\mathbf{p}', t - \tfrac{R}{c}) \frac{\cos\varphi\cos\theta}{\pi R^2} d\Gamma\right)}_{\text{Incident power}}$$

Well-suited method for the determination of time reverberation beyond Sabine's law, (Kuttruff, Miles, Gilbert, Gerlach…)

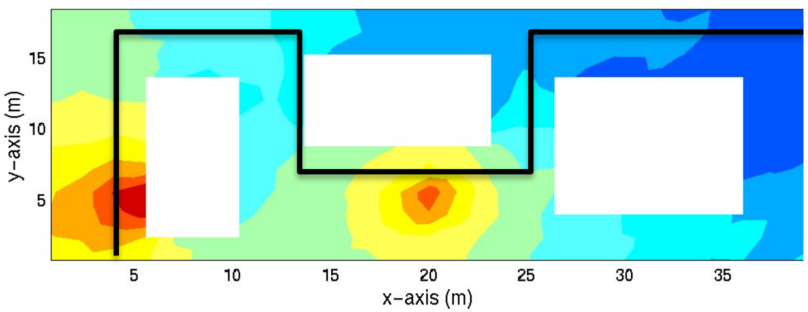
Room acoustics, H. Kuttruff, Elsevier Science. 1973.

Comparison CeReS (CNRS) / RAYON (EDF)

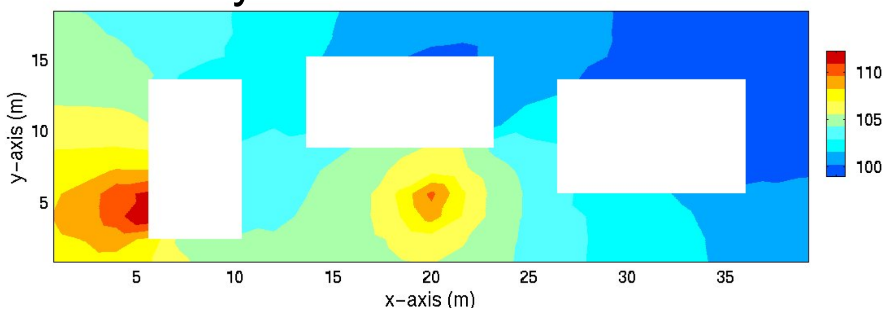




Ray-tracing



Radiosity



Both methods give same results

Comparison of an integral equation and the ray-tracing technique in room acoustics, A. Le Bot, A. Bocquillet J. Acoust. Soc. Am. 2000.

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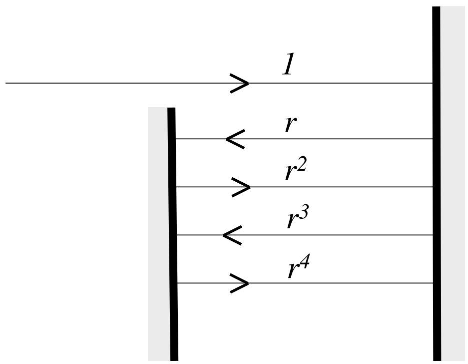
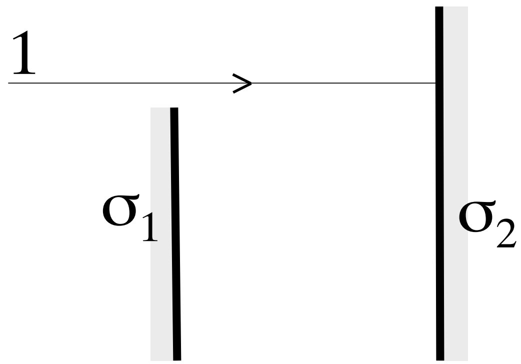
What is the difference between ray-tracing and radiosity?

Ray-tracing / radiative transfer

Ray-tracing method



Radiative transfer method



$$\begin{cases} \sigma_1 = r\sigma_2 \\ \sigma_2 = r(1+\sigma_1) \end{cases} \qquad \begin{cases} \sigma_1 = r^2 / (1-r^2) \\ \sigma_2 = r / (1-r^2) \end{cases}$$

$$W = \sum_{n=0}^{\infty} r^n = \frac{1}{1-r}$$

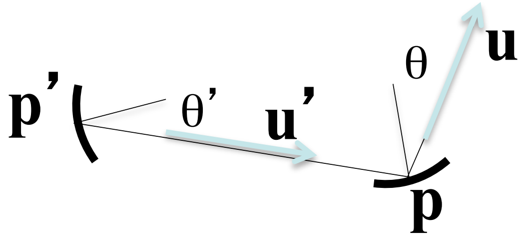
$$W = 1 + \sigma_1 + \sigma_2 = \frac{1}{1-r}$$

Computation of a series

Solving of a linear set

What happens in case of specular reflection?

Specular reflection



$\mathcal{R}$ reflection coefficient
θ, θ' emission angles

$$\mathcal{P}_{\text{ref}} = \mathcal{R}\mathcal{P}_{\text{inc}}$$

Functional equation for reflection

$$\underbrace{\frac{\sigma(\mathbf{p},\mathbf{u},t)}{\cos\theta}}_{\text{Reflected power}} = \underbrace{\mathcal{R}\left[\frac{\sigma(\mathbf{p}',\mathbf{u}',t-\frac{R}{c})}{\cos\theta'} + \int_{\mathbf{p}'\mathbf{p}} \rho(\mathbf{s},t-\tfrac{R}{c})\,dR\right]}_{\text{Incident power}}$$

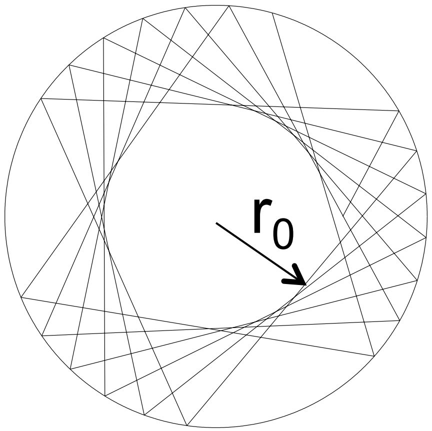
Reflected power

Incident power

Image-source method applies

A functional equation for the specular reflection of rays, A. Le Bot, J. Acoust. Soc. Am. 2002.

Circular billiard

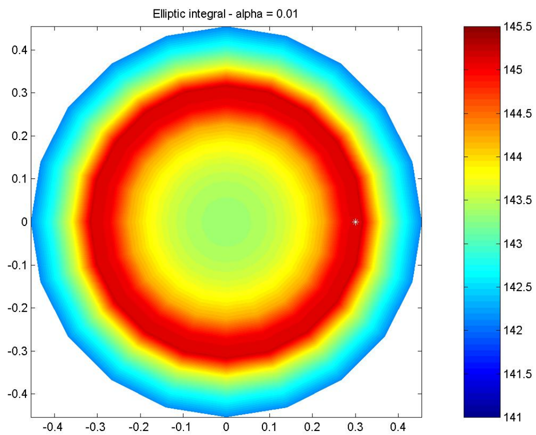


K elliptic integral of the first kind and ε=r_0/r

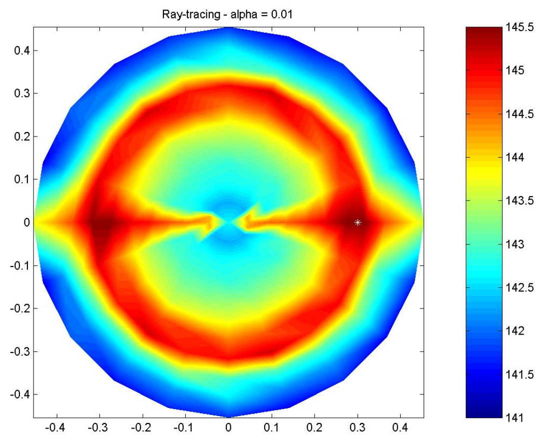
$$W(r) = \frac{4\tau}{\pi c(1-\tau)} \mathrm{K}\left(\frac{1}{\epsilon}\right) \qquad r < r_0$$

$$W(r) = \frac{4\tau}{\pi c(1-\tau)} \epsilon K(\epsilon) \qquad r > r_0$$

Functional equation

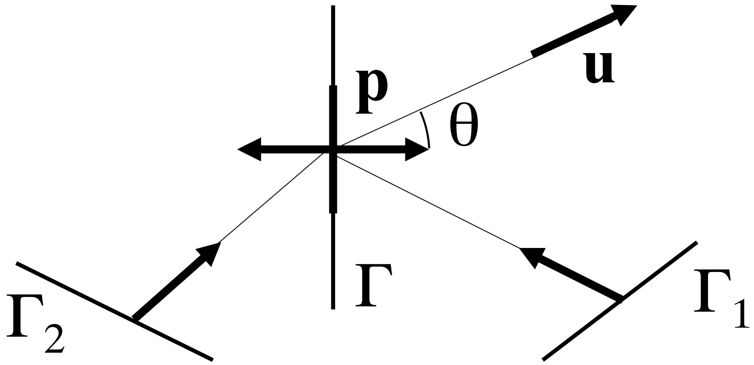


Ray-tracing method



Does it possible to account for wave transmission?

Wave transmission



$\mathcal{R}$ reflection coefficient

$\mathcal{T}$ transmission coefficient

$$\mathcal{P}_{\text{ref}} = \mathcal{R}\mathcal{P}^1_{\text{inc}} + \mathcal{T}\mathcal{P}^2_{\text{inc}}$$

$$\underbrace{\sigma_1(\mathbf{p})}_{\text{Emitted power}} = \mathcal{R}\left(\underbrace{I_1 + \int_{\Gamma_1} \sigma_1 \frac{\cos\varphi\cos\theta}{\pi R^2} d\Gamma}_{\text{Power from }\Gamma_1}\right)$$

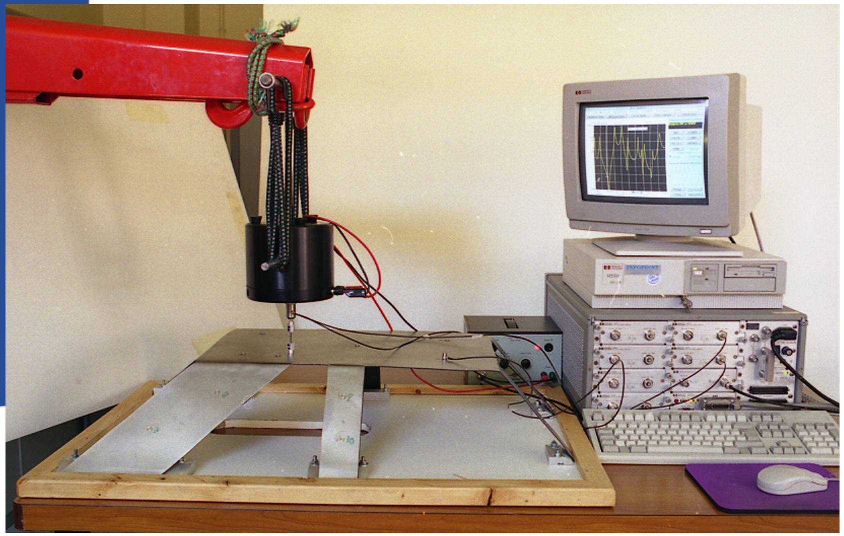
$$+\mathcal{T}\left(\underbrace{I_2 + \int_{\Gamma_2} \sigma_2 \frac{\cos\varphi\cos\theta}{\pi R^2} d\Gamma}_{\text{Power from }\Gamma_2}\right)$$

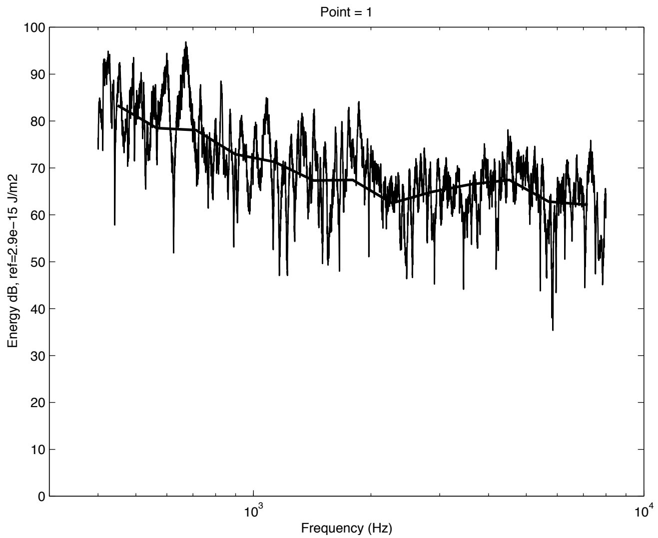
Emitted power

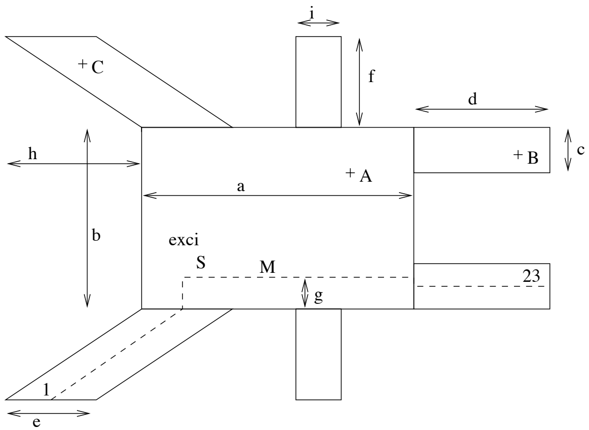
Power from Γ_1

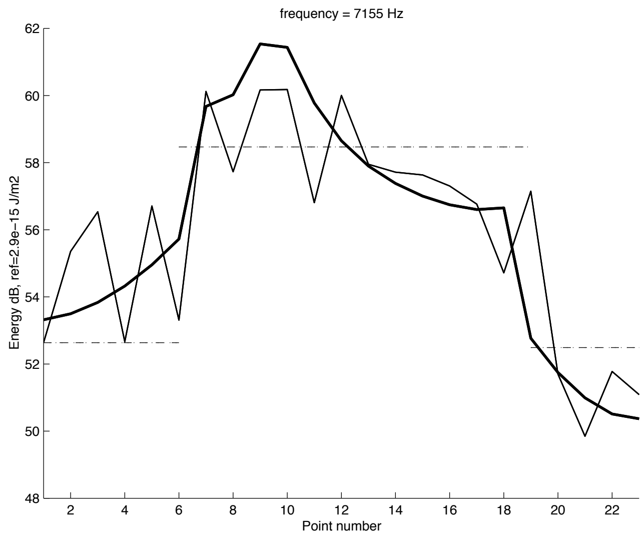
Power from Γ_2

Structural transmission









Energy transfer for high frequencies in built-up structures, A. Le Bot, A. J.Sound Vib. 2002.

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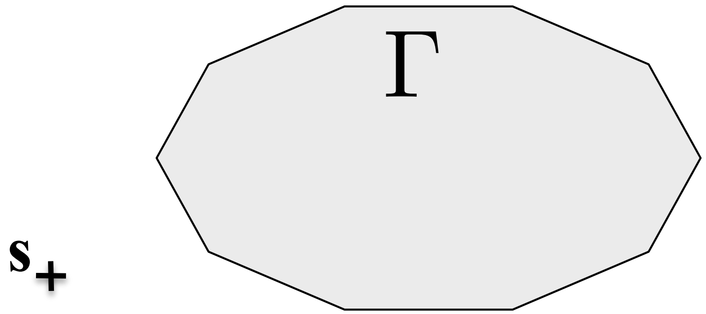
And now diffraction…

Diffraction: The problem

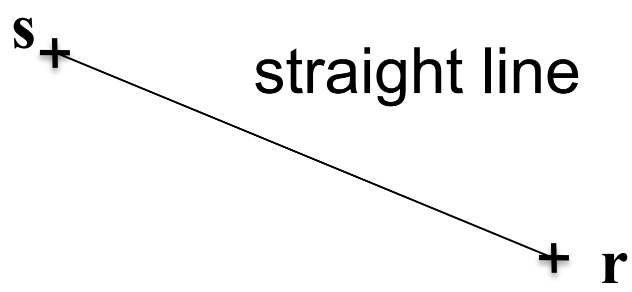


$$\Delta\psi + k^2\psi = -\delta_s$$
$$\partial_n\psi = 0 \text{ on } \Gamma$$

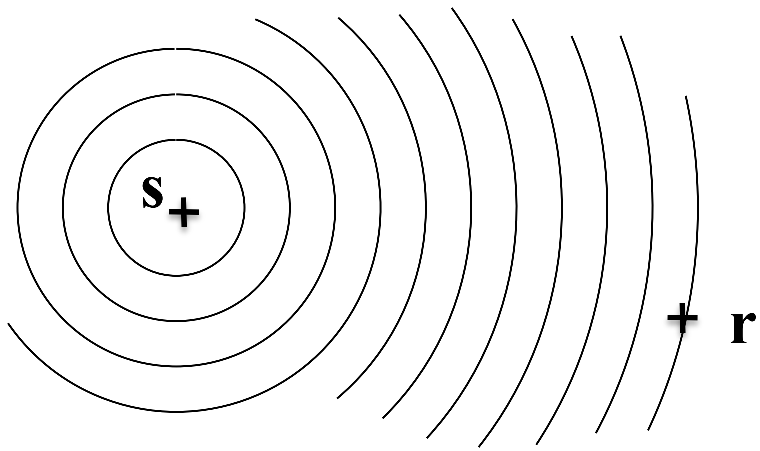
The boundary may be singular

Geometrical acoustics

Ray path



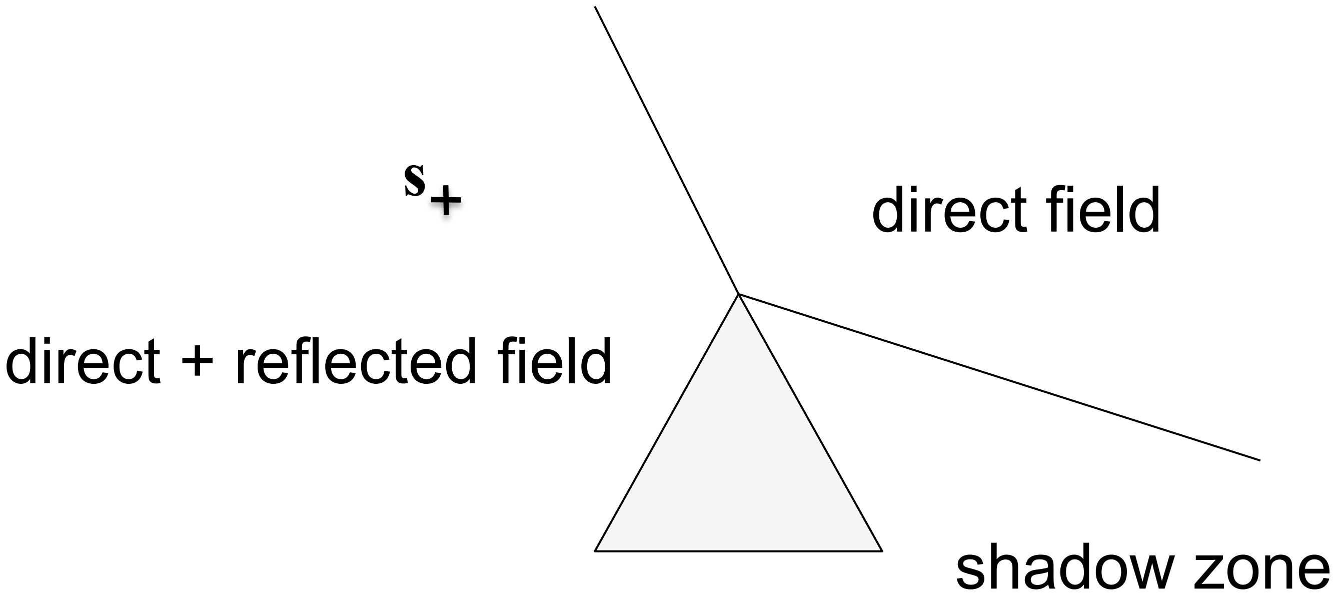
Ray field



$$\psi(\mathbf{r}) = \frac{e^{-ikR}}{\sqrt{R}}$$

$$R = |\mathbf{r} - \mathbf{s}|$$

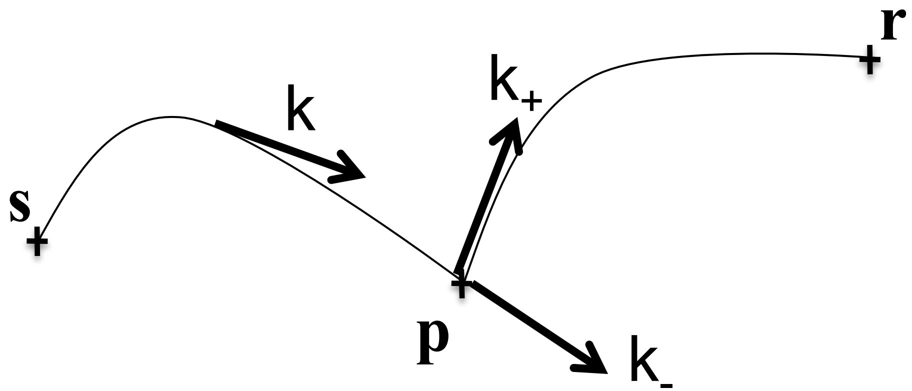
Geometrical acoustics



$\psi = 0$ in the shadow zone!

Generalized Fermat's principle

$$\mathcal{L} = \int k ds \qquad \delta\mathcal{L} = 0$$



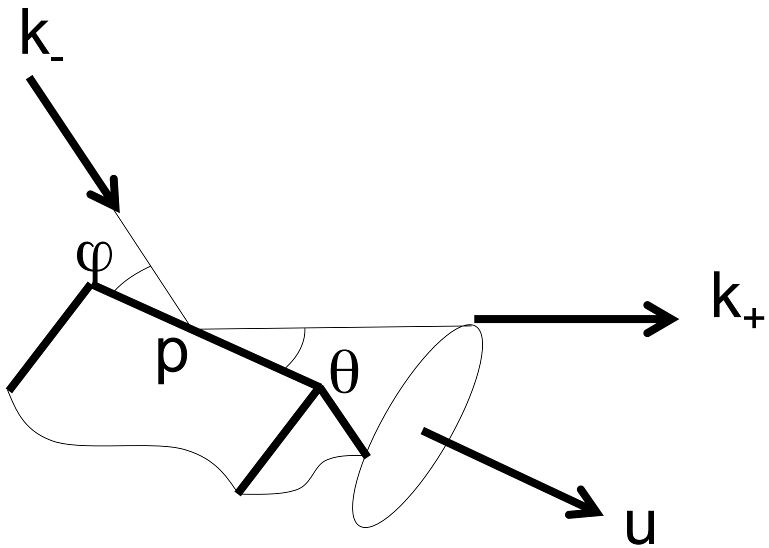
regular path $\delta k = 0$

-> geodesic line (straight line in flat space)

singular path $(k_- - k_+).\delta p = 0$

-> condition on k_- and k_+

Diffracted ray



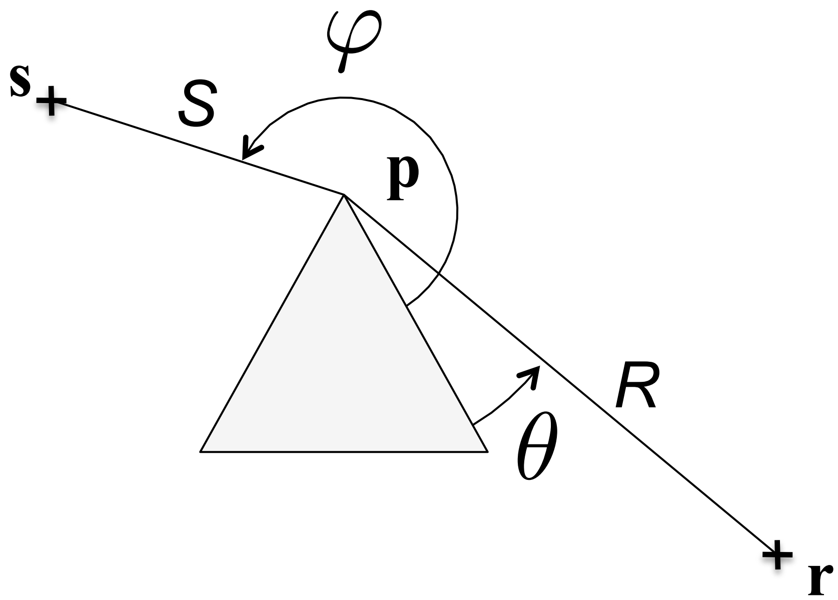
$$(\mathbf{k}_- - \mathbf{k}_+).\delta\mathbf{p} = 0 \qquad \delta\mathbf{p} = \mathbf{u}ds$$

Keller's law

$$\mathbf{k}_-.\mathbf{u} = \mathbf{k}_+.\mathbf{u} \qquad \boxed{\varphi = \theta}$$

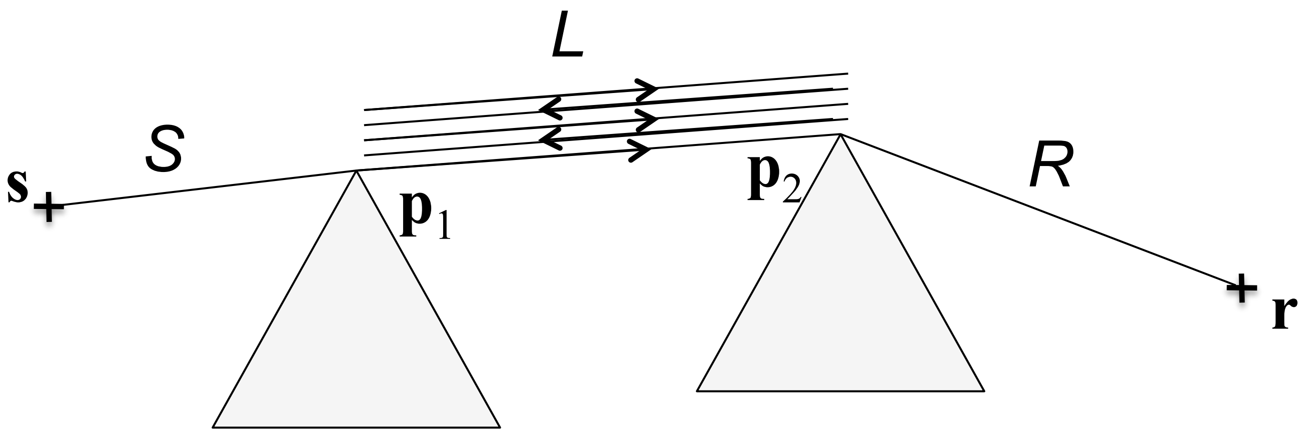
The geometrical theory of diffraction, J.B. Keller, J. Opt. Soc. Am. 1952.

Simple diffraction



$$\psi(\mathbf{r}) = \frac{e^{ikS}}{\sqrt{S}} D(\varphi, \theta) \frac{e^{ikR}}{\sqrt{R}}$$

Double diffraction

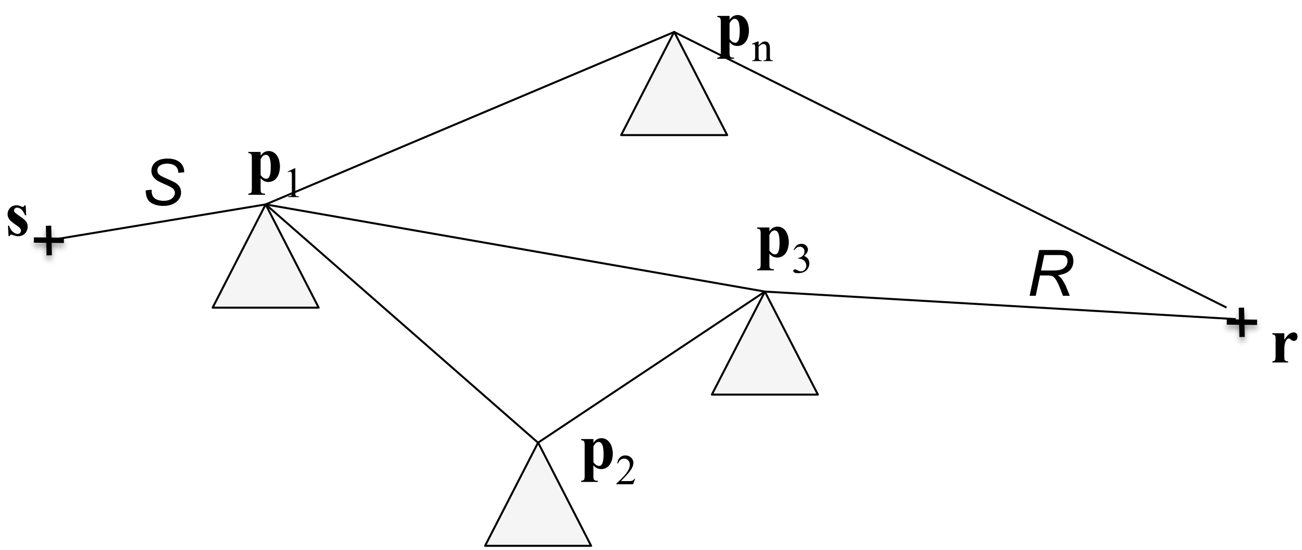


$$\psi(\mathbf{r}) = \frac{e^{ikS}}{\sqrt{S}} D_1' \frac{e^{ikL}}{\sqrt{L}} \times D_2' \frac{e^{ikR}}{\sqrt{R}}$$

$$+ \frac{e^{ikS}}{\sqrt{S}} D_1' \frac{e^{ikL}}{\sqrt{L}} D_1 \frac{e^{ikL}}{\sqrt{L}} D_2 \frac{e^{ikL}}{\sqrt{L}} D_2' \frac{e^{ikR}}{\sqrt{R}}$$

$$+ \dots$$

Multiple diffraction



All ray paths from source to receiver must be determined!

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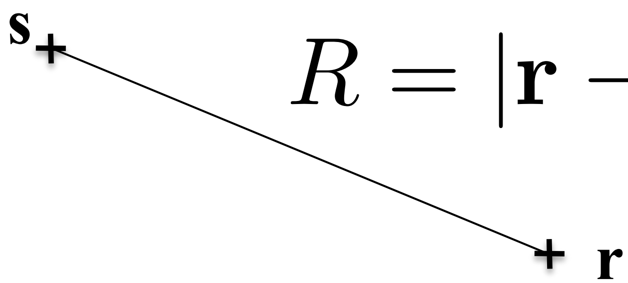
Energy of cylindrical wave



$$R = |\mathbf{r} - \mathbf{s}|$$

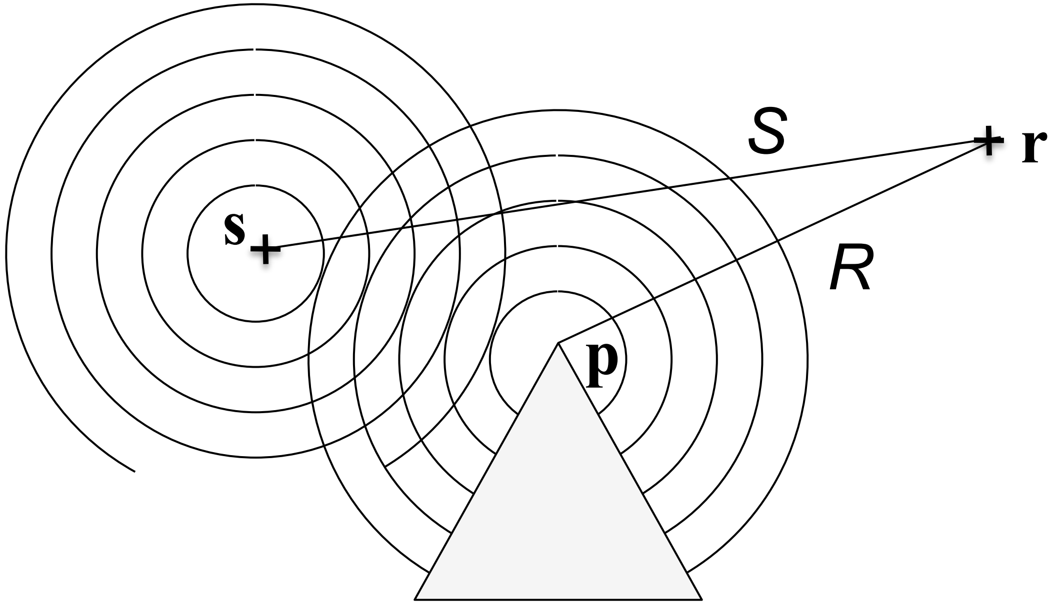
Field

$$\psi(\mathbf{r}) = \frac{e^{-ikR}}{\sqrt{R}}$$

Energy

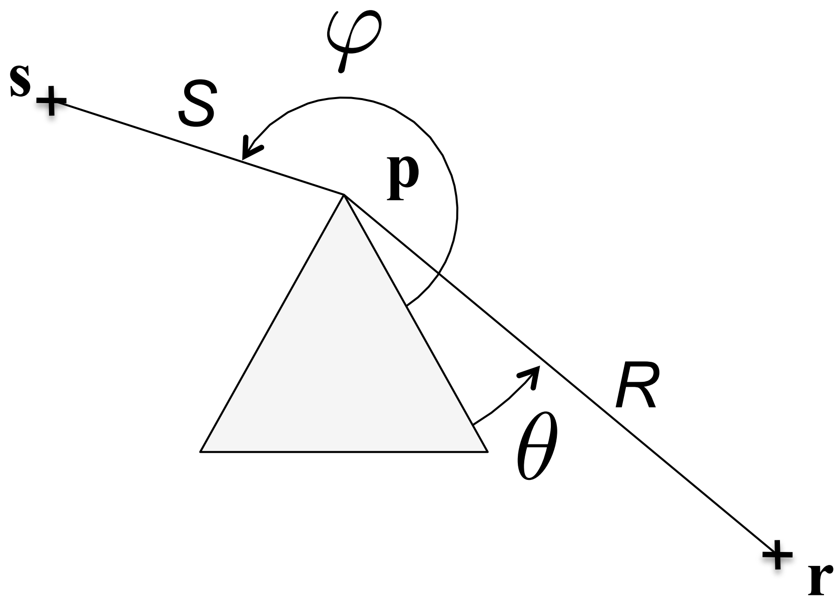
$$W(\mathbf{r}) = \frac{1}{2\pi c R}$$

Diffraction source



$$W(\mathbf{r}) = \frac{1}{2\pi c S} + \frac{\sigma}{2\pi c R}$$

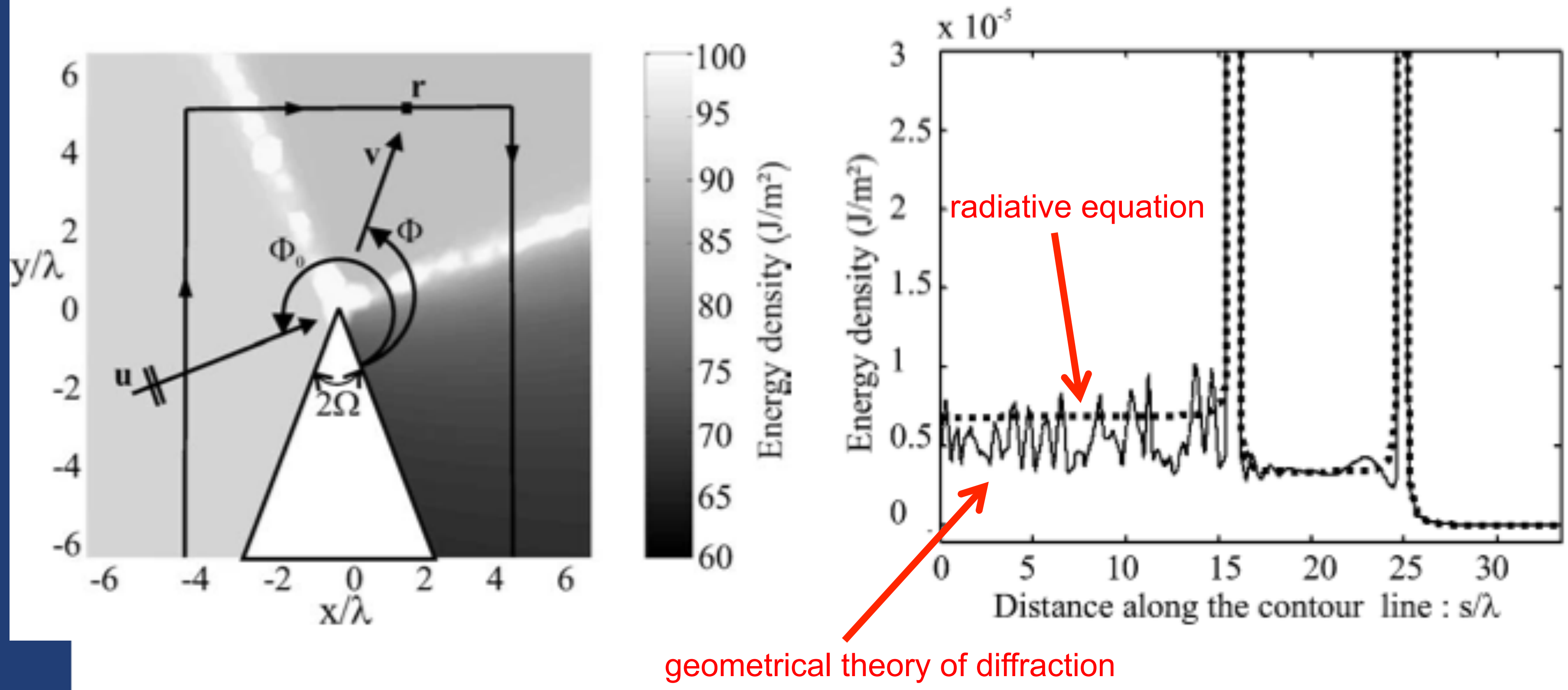
Simple diffraction



$$\frac{\sigma(\theta)}{2\pi} = \underbrace{\frac{1}{2\pi S}}_{\text{Incident intensity}} \times D(\varphi, \theta)$$

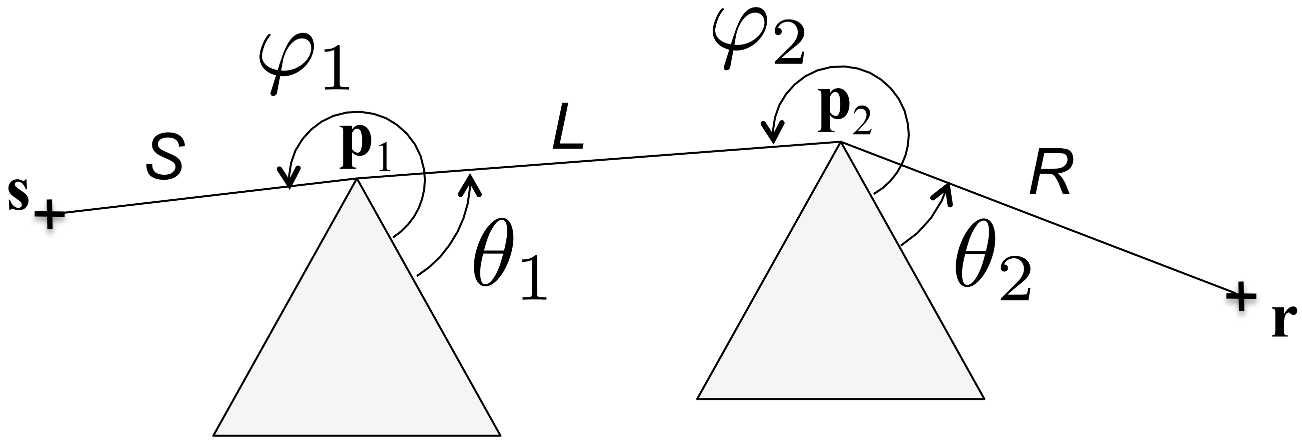
Diffraction efficiency

Simple diffraction



Introduction of acoustical diffraction in the radiative transfer method, E. Reboul, A. Le Bot, J. Perret-Liaudet CRAS 2004.

Double diffraction



Functional equations

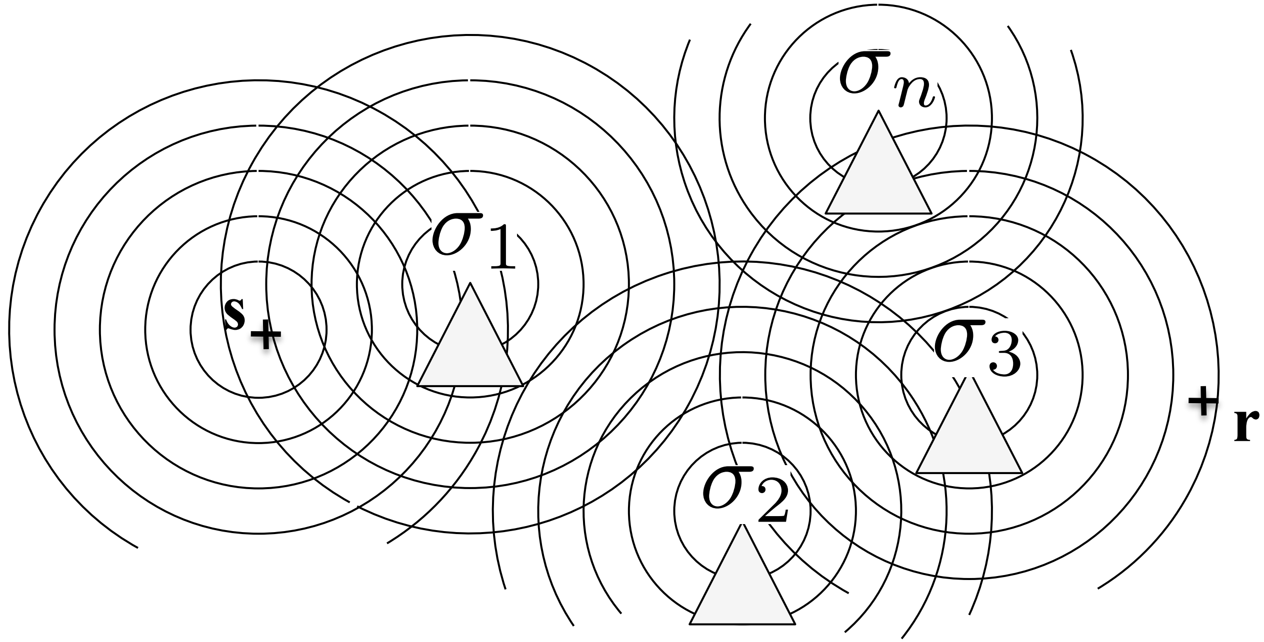
$$\sigma_1(\theta_1) = D(\varphi_1, \theta_1)\frac{1}{S} + D(\theta_1, \theta_1)\frac{\sigma_2(\varphi_2)}{L}$$

$$\sigma_2(\theta) = D(\varphi_2, \theta)\frac{\sigma_1(\theta_1)}{L} \qquad (\theta = \varphi_2)$$

Energy at receiver

$$W(\mathbf{r}) = \frac{\sigma_2(\theta_2)}{cR}$$

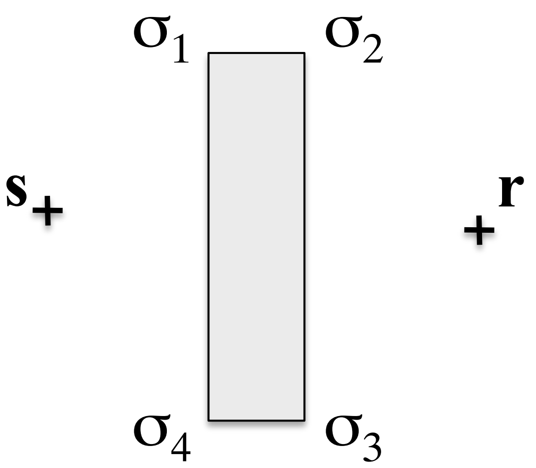
Multiple diffraction



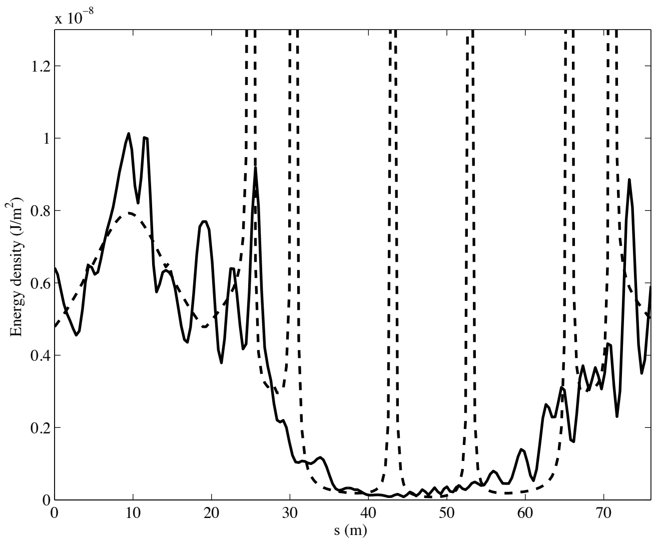
$$\begin{pmatrix} \sigma_1 \\ \vdots \\ \sigma_n \end{pmatrix} = \begin{pmatrix} M \end{pmatrix} \begin{pmatrix} \sigma_1 \\ \vdots \\ \sigma_n \end{pmatrix} + \begin{pmatrix} 1/S \\ \vdots \\ 0 \end{pmatrix}$$

$$W(\mathbf{r}) = \sum_{i=1}^{n} \frac{\sigma_i}{2\pi c R_i}$$

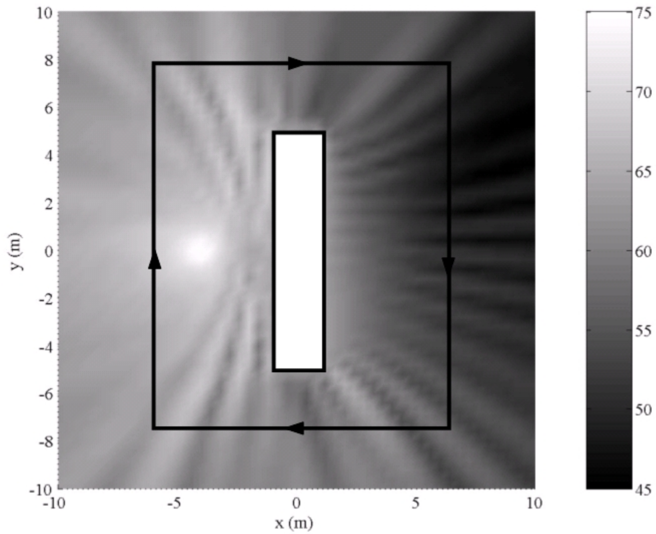
Multiple diffraction



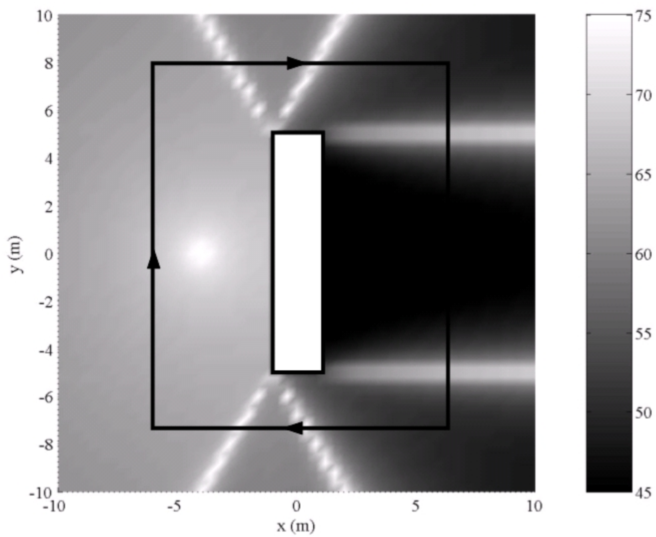
250 Hz



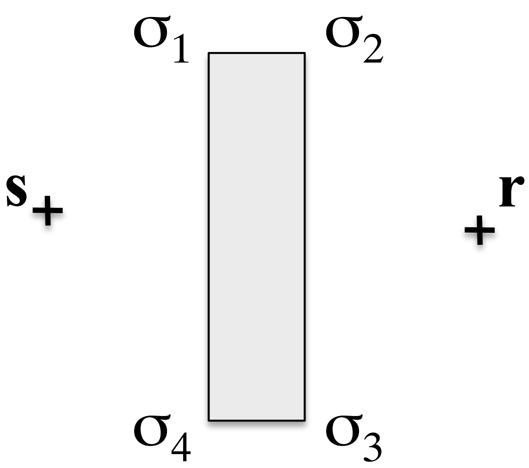
BEM solution



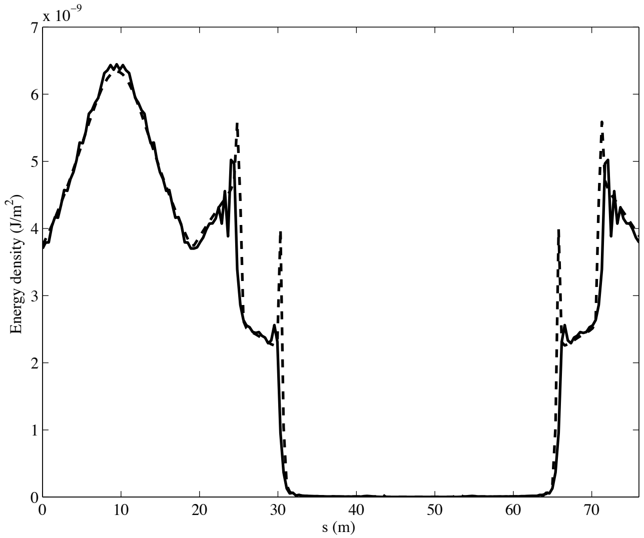
Functional equation



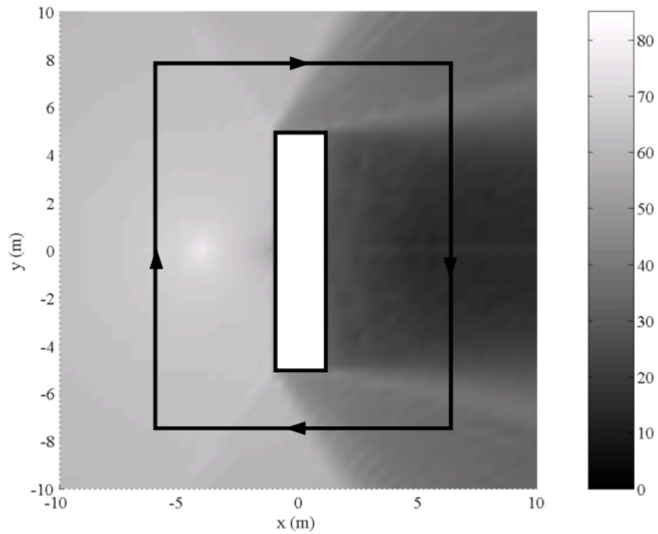
Multiple diffraction



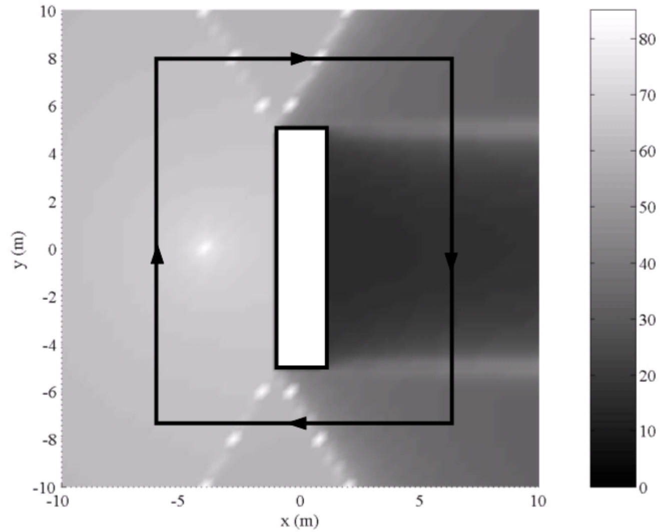
5000 Hz



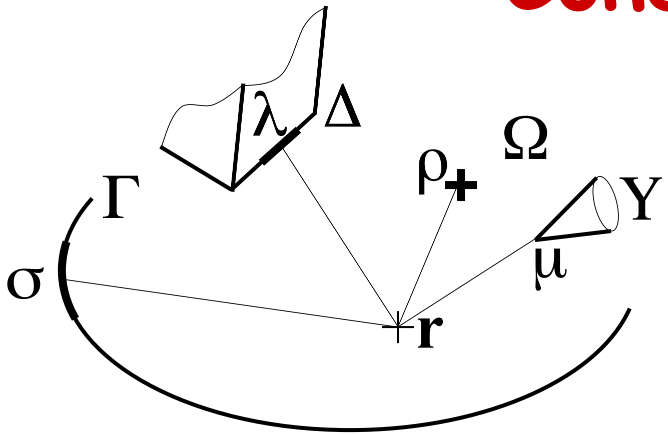
BEM solution



Functional equation



Conclusion



ρ power density of volume sources
σ power density of surface sources
λ power density of line sources
μ power of point sources

$$W(\mathbf{r}) = \underbrace{\int_{\Omega} \rho \frac{d\Omega}{4\pi c R^2}}_{\text{direct field}} + \underbrace{\int_{\Gamma} \sigma \frac{d\Gamma}{4\pi c R^2}}_{\text{reflected field}}$$

$$+ \underbrace{\int_{\Delta} \lambda \frac{d\Delta}{4\pi c R^2}}_{\text{diffracted by edges}} + \underbrace{\sum_{Y} \mu \frac{1}{4\pi c R^2}}_{\text{diffracted by peaks}}$$