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# Integral Formulation for high Frequencies in Assembled Plates

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**Summary:** This paper presents a simplified model for high frequencies in structures. This model wich does not take into account interferencies between propagative waves, is asymptotic in sense that it is more accurate as frequency increases. Based on energetic quantities and energy balance, the spirit of *Statistical Energy Analysis* (SEA) is conserved. But the description is more precise and, in particular, the repartition of energy density inside each sub-system is predicted. Results of experimentation are presented and point out that this model well predicts a frequency average of the measurements.

## 1 INTRODUCTION

Several attempts to generalise the *Statistical Energy Analysis* beyond its limit of application has been performed for few years. Nefske and al [1] proposed a model based on an equation similar to the heat conduction equation. As in SEA, it involves energy quantities: energy and power. But, unlike SEA which involves global variables, this model considers local variables: energy density and energy flow. Many studies [2] have follow this previous work and have pointed out the numerical advantage over finite element method. Moreover, it is possible to reuse existing thermal softwares. However, Langley [3] criticized the application of this method for multi-dimensional structures. He remarked that the direct field predicted by this method decreases slower than the prediction of classical equation of movement. This paradox motivated investigations summarized in [4] where an attempt to explain it is presented.

The purpose of this paper is to present an alternative of the heat conduction equation applied to high frequencies in plates or acoustical enclosures. This formulation, taking into account Langley's remark, is based on an integral formulation deduced from the Huygens principle. The direct field appears explicitly allowing a correction of the heat conduction drawback.

## 2 THEORITICAL FORMULATION

A vibrating system is fully described with two fields: the scalar field of energy density $W$ defined as the sum of kinetic energy density and deformation energy density (for structures) and a vector field of energy flow $\mathbf{I}$ which indicates the direction and the strenght of propagation of energy.

A first set of assumptions required to derive the energy model is summarized as follows:

*(H1) Linear, isotropic, homogeneous two-dimensional system in steady state conditions, excited over the broadband* $[\omega - \Delta\omega/2, \omega + \Delta\omega/2]$,
*(H2) light hysteretic damping loss factor* ($\eta \ll 1$),
*(H3) evanescent waves and near-fields are neglected,*
*(H4) interferencies between propagative waves are not taken into account.*
Another assumption will be added later on.

First, let study the direct field. A pure travelling wave issued from the point source $S$ is caracterized by a proportionality between the energy flow and the energy density: $\mathbf{I}(M) = c_g W(M)\mathbf{u}_{SM}$ where $\mathbf{u}_{SM}$ is the unity vector from $S$ toward $M$ and $c_g$ is the group velocity. By substituting this relationship into the power balance and taking into account the isotropy of space, the following expressions are obtained [4]:

$$G(S,M) = e^{-\frac{\eta\omega}{c_g}SM}/SM \quad \mathbf{H}(S,M) = c_g e^{-\frac{\eta\omega}{c_g}SM}\mathbf{u}_{SM}/SM \tag{1}$$

where $G(S,M)$ denotes the energy density in $M$ of the direct field created by a source $S$ and $\mathbf{H}(S,M)$ denotes the energy flow vector of the direct field. These elementary fields will be oftently used in what follows.

Now, let consider more general fields in a bounded or unbounded domain. Those fields result from a superposition of many travelling waves. As interferencies between travelling waves were declared to be irrelevant in high frequencies, this superposition is considered to be linear. Most general fields are therefore merely constructed by adding elementary fields. Moreover, Huygens principle claims that a general field is a superposition of a direct field emerging from actual sources located in the domain $\Omega$ and a reflected field emerging from fictives sources located on the boundaries $\partial\Omega$ of the domain. Let denote $\rho(S)$ the actual sources at $S$ and $\sigma(P)$ the fictive sources at $P$, complete fields are then:

$$W(M) = \int_\Omega \rho(S)G(S,M)dS + \int_{\partial\Omega} \sigma(P) f\left(\mathbf{u}_{MP}.\mathbf{n}_P\right) G(P,M)dP \tag{2}$$

$$\mathbf{I}(M) = \int_\Omega \rho(S)\mathbf{H}(S,M)dS + \int_{\partial\Omega} \sigma(P) f\left(\mathbf{u}_{MP}.\mathbf{n}_P\right) \mathbf{H}(P,M)dP \tag{3}$$

The function $f$ appearing in (2,3) is the directivity diagramm of the secondary source $\sigma$. It depends on the angle between the considered direction $\mathbf{u}_{MP}$ and the inward normal $\mathbf{n}_P$ at the point $P$. It is subjected to the additional assumption:

*(H5) Fictive sources have all the same directivity $f$ which does not depend on the point $P$ and chosen respect to Lambert law* $f(\mathbf{u}.\mathbf{n}) = \mathbf{u}.\mathbf{n}$.

Obviously, actual sources can be considered to be known. In opposition, fictive sources are unknown and an additional equation must be derived to determine it. This equation is obtained by applying local power balance at a point $P$ located on the boundary. It yields:

$$\gamma c_g \sigma(P) = \left\{ \int_\Omega R\rho(S)\mathbf{H}(S,P)dS + \int_{\partial\Omega} R\sigma(P')f\left(\mathbf{u}_{PP'}.\mathbf{n}_{P'}\right)\mathbf{H}(P',P)dP' \right\}.\mathbf{n}_P \quad (4)$$

where $\gamma$ is a constant depending on the choice of $f$. For Lambert law $\gamma = 2$ in two dimensions. $R$ is the reflection efficiency of the boundary at $P$. For undamping boundary $R = 1$. This equation (4) is a Fredholm integral equation of second kind.

In another hand, an interface between two different media requires two integral equations to determine fictive sources on each side of this interface. Let caracterize the energy transfer occuring with two ratios: the reflection efficency $R$ which is the ratio of reflected power over incident power and the transmission efficency $T$ which is the ratio of transmitted power over incident power. Obviously, the sum of these efficencies equals to one for non dissipative interface. Note that these efficencies may depend on the incident direction. Two power balances can now be stated at a given point $P$ on the interface: the power emmited toward medium one is the sum of the reflected part of the incident power coming from medium one and the transmitted part of the incident power coming the medium two, and so on. It yields:

$$\begin{aligned} &\gamma c_{g_1}\sigma_1(P) = \\ &\left\{\int_{\Omega_1} R\rho(S)\mathbf{H}_1(S,P)dS + \int_{\partial\Omega_1} R\sigma_1(P_1')f\left(\mathbf{u}_{PP_1'}.\mathbf{n}_{1,P_1'}\right)\mathbf{H}_1(P_1',P)dP_1'\right\}.\mathbf{n}_{1,P} + \\ &\left\{\int_{\Omega_2} T\rho(S)\mathbf{H}_2(S,P)dS + \int_{\partial\Omega_2} T\sigma_2(P_2')f\left(\mathbf{u}_{PP_2'}.\mathbf{n}_{2,P_2'}\right)\mathbf{H}_2(P_2',P)dP_2'\right\}.\mathbf{n}_{2,P} \end{aligned} \quad (5)$$

$$\begin{aligned} &\gamma c_{g_2}\sigma_2(P) = \\ &\left\{\int_{\Omega_1} T\rho(S)\mathbf{H}_1(S,P)dS + \int_{\partial\Omega_1} T\sigma_1(P_1')f\left(\mathbf{u}_{PP_1'}.\mathbf{n}_{1,P_1'}\right)\mathbf{H}_1(P_1',P)dP_1'\right\}.\mathbf{n}_{1,P} + \\ &\left\{\int_{\Omega_2} R\rho(S)\mathbf{H}_2(S,P)dS + \int_{\partial\Omega_2} R\sigma_2(P_2')f\left(\mathbf{u}_{PP_2'}.\mathbf{n}_{2,P_2'}\right)\mathbf{H}_2(P_2',P)dP_2'\right\}.\mathbf{n}_{2,P} \end{aligned} \quad (6)$$

The set of integral equation (4,5,6) may be solved with an appropriate numerical scheme. For instance, in the following application, a collocation method with constant elements has been retained. As a second step, the fields $W$ and $\mathbf{I}$ may de constructed involving equations (2,3).

## 3 APPLICATION TO ASSEMBLED PLATES

Previous method is implemented as a software to solve integral equations in case of assembled plates. The reflection and transmission efficiencies depends both on the angle between the coupled plates and the incident angle. Those efficiencies and other caracteristics such as group velocities are evaluated on the base of classical Love plate model. The results of this model has been tested with experimental measurements realized on a seven plates structure. This structure is shown in Fig.1. It was excited on the top with a shaker and acceleration was measured on several

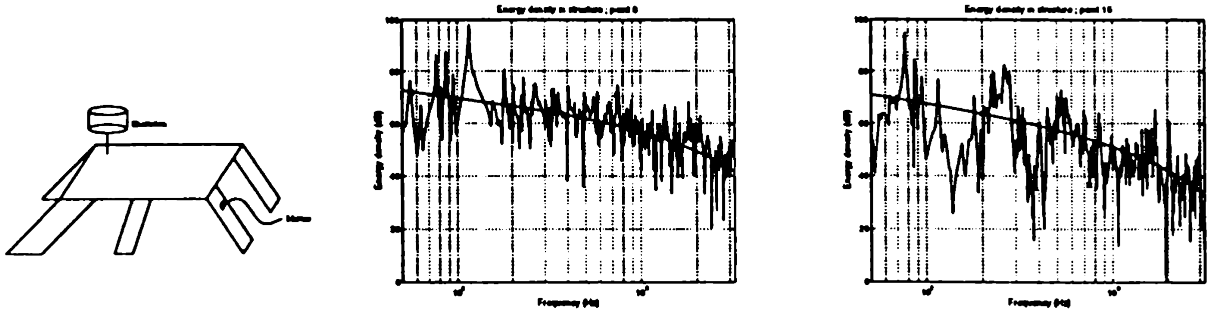

Figure 1: Comparison: x, measurement - o, prediction

points distributed on the top and the sides. The measured energy density was evaluated as being twice the kinetic energy density proportional to the square of the FRF. Note that the phase information is not necessary in this calculation. A comparison between measured and predicted energy densities is shown in Fig.1 at two points. The first one is located in the vicinity of the excitation point and the second in the side. The prediction well estimates a frequency average of the measured response. All detail relevant to the modal behavior of the structure disapear in the energetic model.

## 4 CONCLUSIONS

In this study, a simplified model well suited for high frequencies has been presented. This model is based on an approximation related to high frequencies: interferencies between travelling has not been taken into account. In other words, the modal behavior of the structure is considered to be irrelevant. Comparisons with measurements point out that results of this method have to be interpreted as frequency average over frequency.

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