INTERNATIONAL CONGRESS ON
RECENT DEVELOPMENTS IN AIR- AND
STRUCTURE-BORNE SOUND AND VIBRATION
MARCH 6-8, 1990 AUBURN UNIVERSITY, USA

NUMERICAL DETERMINATION OF SOUND SOURCE RADIATION USING NEAR FIELD INTENSITY IN LOW FREQUENCY RANGE

Emile LUZZATO and Alain LEBOT
Département AMV - DER - EDF, 1 Avenue du Général de Gaulle
92141 - CLAMART CEDEX - FRANCE

ABSTRACT

Industrial sound sources have complex radiation characteristics and therefore their description by means of numerical simulation can only be made with sophisticated numerical models. In the low frequency ranges, these models must be able to represent accurately the diffraction phenomena whose effects are very important on the radiated sound field. This article presents reconstruction methods of the sound source radiation characteristics represented by the acoustic power of its distinctive elements. A reconstruction method consists of identifying the parameters of a given numerical model used to simulate the source radiation. The results of the reconstruction are presented for various near field given data used in the identification process such as complex sound pressures and complex sound intensities. Various sound source configurations have been studied with various radiation models. The most interesting results were obtained with a finite difference model which allowed the computation of the sound field in a given enclosure. Identification techniques based on various minimization functionals have also been tested. In general, the results show the identification made with sound intensity is far superior than that made with sound pressure data. Moreover, when the simulation model only takes into account the geometrical characteristics of the source without consideration of the environment such as the other machines and/or the enclosure boundaries, the identification made with pressure data is quite inaccurate whereas the identification made with sound intensity remains acceptable.

INTRODUCTION

Many fields of physics such as acoustics are concerned with the analysis of systems in which fields are generated by external sources. Solving the subsequent equations generally requires a sophisticated computer model. The function of this model is very simple : for a given set of sources, of geometrical parameters and/or of boundary conditions, it allows one to determine the fields generated by the given set of sources in their definition domains. For most of this type of problems, the data being measured are only field data and there is often little knowledge about the sources. The identification process is a mathematical tool [1] which enables one to find more informations on these elements that are not directly accessible with measurements, for example : the sources.

Let a sound field generated by sound sources be measured at discrete points of a definition domain. Let also the sound sources be represented by parameters and let the computer model be able to determine the resulting acoustic field. The identification consists of finding the source parameters that generate a sound field which most closely matches the measured sound field. In other words, a real set of sound sources can be identified to a modelled set of sound sources if they generate similar sound fields. Four elements are important in this approach :

- the parametric representation of source radiation ,
- the nature of the measured data in the sound field,

- the numerical model that links the source parameters to the measured data,
- the mathematical process of optimal identification of the sources parameters.

In this study, the measured data used to represent the sound field for sound source identification purposes are :

- the acoustic pressure,
- the acoustic intensity.

These data allow one to partially describe the sound field, but however lead to different identification results due to their different nature. This particular aspect of sound source identification will be emphasized in this paper.

POINT SOURCES IN FREE FIELD

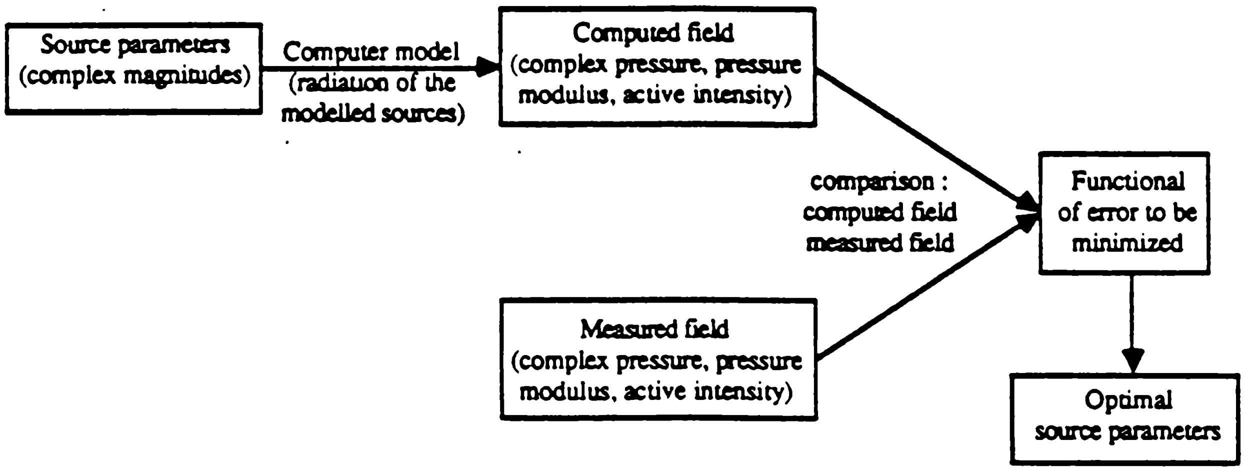
Identification Principles

In order to understand the principles of the sound source identification process, it is tested in the simple case of point sources radiation. The question is the following : is it possible to reproduce the sound field generated by a given set of point sources by means of another set of point sources ? The corollary to this question is : what should one measure to obtain the best sound field reproduction and therefore the best sound source identification ?

Each of the considered point source in free field is defined by its coordinates and its complex magnitude. The considered source parameter used in the identification is its complex magnitude. Three kinds of measurements are supposed to be accessible :

- the complex acoustic pressure,
- the modulus of the complex acoustic pressure,
- one of the components of the active intensity vector.

For each kind of measurement, one should expect a specific identification result in terms of source magnitude. The following diagram illustrates the identification principle.



The mathematical properties of an error functional depend upon the nature of the measured field. In particular, on shows that the error functional defined from complex acoustic pressures is convex whereas the functional defined with acoustic intensity is not. In each case, one should use an appropriate minimization algorithm. If the functional is convex, a classical generalized inversion technique can be used. On the other hand, if the functional is not convex, one should set up constraints on the minimization technique and therefore use an iterative process [2,3,4].

In this numerical study, the measured acoustic field is in fact a computed field called a reference field that results from a given set of sound sources called reference sources. Therefore, there are two kinds of fields : the reference field and the identification field, and there are two kinds of sources : the reference sources and the identification sources.

Point Source Model

The exact solution of the Helmholtz equation :

$$(\Delta + k^2)\,p = \sum_{j=1}^{n} A_j\,\delta(M_j)$$

for a given set of n_s point sources with complex magnitudes A_j is the following :

$$p(M) = \sum_{j=1}^{n_s} A_j \frac{e^{i k r(M-M_j)}}{r(M-M_j)}$$

The complex acoustic intensity vector generated by this point source distribution is :

$$\vec{I}(M) = \mathrm{Re}\left[-\frac{i}{2\rho\omega}\left(\sum_{j=1}^{n_s} A_j \frac{e^{ikr(M-M_j)}}{r(M-M_j)}\right)\left(\sum_{j=1}^{n_s} A_j^* \frac{e^{-ikr(M-M_j)}}{r^2(M-M_j)}\left(ik - \frac{1}{r(M-M_j)}\right)\vec{r}(M-M_j)\right)\right]$$

The chosen excitation frequency is 340 Hz so that the wavelength λ is 1 m. The space domain in which the point sources are distributed has a characteristic dimension of half a wavelength. The reference sound field is computed for 6 reference points sources arbitrarily distributed in this domain. The identification sound fields are obtained after minimization of a given functional, for 8 identification point sources arbitrarily distributed in this domain. The n_p=104 measurement points are regularly distributed at a spherical surface, centered at the average center of the reference source distribution. The radius of this measurement spherical surface is 2 m, that is 2λ.

Error Functional Minimization

Two different error functionals, expressed in terms of the complex magnitude vector A of dimension n_s=8, have been defined. The first one $\phi_p(A)$ represents the averaged error between the two pressure squared modulus fields, the reference field p^r and the identification field p^i :

$$\phi_p(A) = \sum_{k=1}^{n_p}\left[(p_k^i(A))^*\,p_k^i(A) - (p_k^r)^*\,p_k^r\right]^2$$

The second one $\phi_I(A)$ represents the averaged error between the two normal active intensity fields, the reference field I^r and the identification field I^i :

$$\phi_I(A) = \sum_{k=1}^{n_p}\left[\,I_k^i(A) - I_k^r\,\right]^2$$

These two functionals are not convex, and therefore they may possess many minima. The same initialization vector A is chosen for both minimizations. The minimization algorithm used is the so called "trust region method" [3]. It has the great advantage of stability. It always finds the minimum of a given non convex functional that is the closest from the initial vector A. It is inspired from known gradient methods such as conjugate gradient or quasi-Newton methods. Its principle consists in finding a local quadratic approximation, tangent to the functional at the current iteration point for each iteration. The size of the trust region is then defined by linear constraints according to the quality of the local approximation : the better the quality of the approximation, the larger the trust region.

Numerical Results

Figure 1 shows the convention used for the representation of the sound pressure level observed at the measurement spherical surface. The sphere is developed on a rectangular plane so that its two polar points are represented by the lower and the upper horizontal lines.

Figures 2 represents the sound pressure levels observed on the spherical surface for the 6 reference point sources. Figure 3 represents the identification sound pressure levels calculated after minimization of the pressure error functional $\phi_p(A)$. Figure 4 represents the identification sound pressure levels calculated after minimization of the intensity error functional $\phi_I(A)$. The interesting aspect of this result is that the minimization of the active intensity functional leads to a better representation of the reference source radiation than the minimization of the pressure functional. In this numerical test, the first identification process that tries to reconstruct a sound pressure modulus field leads to a wrong radiation pattern in terms of sound pressure levels, whereas an identification process that tries to reconstruct an intensity field leads to a good approximation of the reference radiation pattern. If this remark is valid in a free field configuration, it will also be true when non free field configurations are considered as shown in the next part.

SOURCE RADIATION IN AN ENCLOSURE

The Numerical Model

A finite difference approximation is used to solve the Helmholtz equation. The enclosure is rectangular and the source is represented by a rectangular block with constant velocity magnitudes on each of its six faces. The dimension n_s of the source vector A, denoted $[A]^{n_s}$, is therefore equal to 6. To solve the Helmholtz equation using discretization techniques such as finite difference, one must solve the following linear system of equations [5] :

$$[H]_n^n \, [P]^n \; = \; ({}^tE_p \,, E_v)_{n_s}^n \, [A]^{n_s}$$

where n is the overall dimension of the discretization and P represents the coordinates of the acoustic pressure field in the discretization basis. [H] is the matrix representation of the Helmholtz operator in finite difference approximation ; it is a square matrix with n rows and n columns. E_p represents the set of the n basis vectors of the pressure fields vector space. E_v represents the set of the n_s basis vectors of the velocity fields at the surface of the source. For each independent unit basis vector v_j, in the set E_v, one can find an elementary solution P_j of the Helmholtz equation such as :

$$[H]_n^n \, [P_j]^n \; = \; ({}^tE_p \,, v_j)^n$$

Let $[\underline{P}_o]$ be the matrix defined by :

$$[\underline{P}_o]_{n_s}^{n_p} \; = \; [\underline{H}^{-1}]_n^{n_p} \, ({}^tE_p \,, E_v)_{n_s}^n$$

where $[\underline{H}^{-1}]$ is issued from the inverse of the Helmholtz operator matrix in which only the rows relative to the observation points have been kept. Therefore, $[\underline{P}_o]$ represents the elementary complex sound pressures at the n_p observation points and the sound pressure resulting from a given magnitude vector $[A]^{n_s}$ is :

$$[\underline{P}]^{n_p} \; = \; [\underline{P}_o]_{n_s}^{n_p} \, [A]^{n_s}$$

This last matrix relation is the source model associated to the identification problem. This source model that has been established with a finite difference approximation could also be defined with other kinds of approximation methods such as a finite elements or boundary elements methods. All the geometrical characteristics of the enclosure as well as the acoustic boundary conditions such as impedance conditions, are included in the source model.

From the source model giving the acoustic pressure, one can express the acoustic intensity at point k of the observation surface :

$$I_k \; = \; \left[[\underline{P}_{o_k}]_{n_s} [A]^{n_s}\right] \left[[\underline{Q}_{o_k}]_{n_s} [A]^{n_s}\right]^*$$

where $[\underline{Q}_o]$ is a matrix derived from the gradient of matrix $[\underline{P}_o]$ components.

The reference complex pressures and the reference normal active intensities are computed at a distance of $\lambda/10$ from the source surface (figure 5) in six distinct planes parallel to the six faces of the rectangular source. The distances between the source surfaces and the corresponding walls of the enclosure are roughly equal to one wavelength.

In the various sound source identifications, the parameter that changes the acoustic configuration in the computation of the error between the identification source model and the reference source model is the average absorption factor α. For each numerical test, the impedances of the enclosure walls are defined so that each wall has the same plane wave absorption factor α :

$$\alpha = \frac{4\,Z_r}{(Z_r+1)^2 + (Z_i)^2} \qquad \text{where} \qquad \begin{array}{l} Z_r = \mathrm{Re}\,(Z) \\ Z_i = \mathrm{Im}\,(Z) \end{array}$$

The difference between the reference absorption factor and the identification absorption factor allows one to represent the distance between the reference model and the identification model. In the identification source model, the average absorption factor is always 1 so that it gives a rough approximation of the free field radiation.

Error functional minimization

Three functionals are being considered.

a - The complex acoustic pressure functional : The first functional is quadratic in terms of complex acoustic pressures ; it is therefore convex in terms of the vector [A] of complex sources magnitudes :

$$e_a(A) = \left[[P_1]_{n_s}^{n_p}[A]^{n_s} - [P_o]_{n_s}^{n_p}[A_o]^{n_s}\right]^* \left[[P_1]_{n_s}^{n_p}[A]^{n_s} - [P_o]_{n_s}^{n_p}[A_o]^{n_s}\right]$$

The reference complex pressures are computed for a given vector $[A_o]$ of sources magnitudes and for a given source model $[P_o]$ with a given average absorption factor in the domain [0.3, 1.0]. The identification complex pressures are computed for a current vector [A] of sources magnitudes and for a given source model $[P_1]$ with an averaged absorption factor of 1. The minimization of this functional is realized with a mean least square inversion method. A direct algorithm is used to solve the corresponding linear system of equations :

$$[A]^{n_s} = \left[[P_1^*]_{n_p}^{n_s} [P_1]_{n_s}^{n_p}\right]^{-1} [P_1^*]_{n_p}^{n_s} [P_o]_{n_s}^{n_p}[A_o]^{n_s}$$

b - The active acoustic intensity functional : The second functional expresses the error between the reference and the identification intensity fields ; it is not convex in terms of the vector [A] of complex sources magnitudes and its minimum is not unique :

$$e_b(A) = \sum_{k=1}^{n_p} \left[\left[[P_1]_k\right]_{n_s} [A]^{n_s}\right] \left[\left[[Q_1]_k\right]_{n_s} [A]^{n_s}\right]^* - \left[\left[[P_o]_k\right]_{n_s} [A_o]^{n_s}\right] \left[\left[[Q_o]_k\right]_{n_s} [A_o]^{n_s}\right]^* \right]_{Re}^2$$

Only the real part of the intensity is squared in this error functional. The reference active intensities are computed for a given vector $[A_o]$ of sources magnitudes and for a given source model represented by $[P_o]$ and $[Q_o]$ with a given averaged absorption factor in the domain [0.3, 1.0]. The identification active intensities are computed for a current vector [A] of sources magnitudes and for a given source model represented by $[P_1]$ and $[Q_1]$ with an averaged absorption factor of 1. The minimization of this functional is realized with an iterative technique : the trust region method.

c - The active acoustic intensity functional in terms of $[AA^*]$: The third functional is defined in terms of vector $[AA^*]$ whose dimension is n_s^2 and whose general term is : $(AA^*)_m = A_i A_j^*$ with $m = (i-1)\, n_s + j$. As for error functional e_b, it expresses the error between the reference and the identification intensity fields ; however, it is convex in terms of vector $[AA^*]$ and its minimum is unique :

$$e_c(AA^*) = Re\left[[S_1]_{n_s^2}^{n_p}[AA^*]^{n_s^2} - [S_o]_{n_s^2}^{n_p} [AA_o^*]^{n_s^2}\right]^* Re\left[[S_1]_{n_s^2}^{n_p}[AA^*]^{n_s^2} - [S_o]_{n_s^2}^{n_p} [AA_o^*]^{n_s^2}\right]$$

Only the real part of the intensity is squared in this error functional. Matrices $[S_o]$ and $[S_1]$ are derived from matrices $[P_o]$ and $[Q_o]$, and $[P_1]$ and $[Q_1]$. The reference active intensities are computed for a given vector $[AA^*_o]$ and for a given source model represented by $[S_o]$ with a given average absorption factor in the domain [0.3, 1.0]. The identification active intensities are computed for a current vector $[AA^*]$ and for a given source model represented by $[S_1]$ with an average absorption factor of 1. The minimization of this functional is realized with a mean least square inversion method. A direct algorithm is used to solve the corresponding linear system of equations :

$$[AA^*]^{n_s^2} = \left[[S_1^*]_{n_p}^{n_s^2} [S_1]_{n_s^2}^{n_p}\right]^{-1} [S_1^*]_{n_p}^{n_s^2} [S_o]_{n_s^2}^{n_p} [AA_o^*]^{n_s^2}$$

The dimension of this linear system can often be quite large due to the change of the minimization variables.

Numerical Results

The acoustic power vectors $[W_a]^{n_s}$, $[W_b]^{n_s}$ and $[W_c]^{n_s}$ obtained for the three functional minimizations are compared to the reference power vector $[W_o]^{n_s}$. Four distinct vectors of the acoustic power of the source surfaces are then computed for each considered value of the averaged absorption factor α. The reference power vector $[W_o]^{n_s}$ is determined merely by computing the integral of the intensity model defined by matrices

[$\underline{P}_o(\alpha)$] and [$\underline{Q}_o(\alpha)$]. The others power vectors are associated to the three error functional minimizations, namely :

- error a : $e_a(A)$ - error b : $e_b(A)$ - error c : $e_c(AA^*)$ -

They are computed after the minimization of the corresponding error functional, using the components found for vector (A) with error a and b and for vector (AA^*) with error c. To compute the two vectors $[W_a]^{n_s}$ and $[W_b]^{n_s}$, matrices [$\underline{P}_1$] and [$\underline{Q}_1$] are used ; matrix [$\underline{S}_1$] is used to compute vector $[W_c]^{n_s}$. The computed components of these power vectors are compared to the components of the reference power vector $[W_o]^{n_s}$ in figures 6, 7 and 8, respectively associated to the following values of α : 0.9, 0.6 and 0.3. Figure 9 presents the total acoustic power of the source determined for the reference configuration and for the error functional minimization a, b and c, in terms of the various values of the averaged absorption factor α : 1.0, 0.9, 0.8, 0.6 and 0.3. This last illustration clearly shows the poor accuracy obtained by minimizing error functional a, due to the large differences between matrices [$\underline{P}_o(\alpha)$] and [$\underline{P}_1$] particularly when α is 0.3. When α is 1, all the identification processes lead to the right reference powers as shown in figure 9. When α is less than 1, the acoustic power of the source surfaces can be determined with an acceptable accuracy by minimizing error functional b and c, particularly for the surfaces having large reference powers. In figure 8, the acoustic power found for the surface source 2 with the minimization of error c is 13 dB less than the reference power. This result is due to the small value of the reference power of this surface compared to the power of the other surfaces. The main conclusion of this numerical test is that it is possible to estimate the acoustic power radiated by machine elements in low frequency ranges (when λ is larger than or of the same order as the elements size) when active acoustic intensity measurements are used with a model simulating the acoustic free field radiation of the machine.

CONCLUSION

The main observation of this study is the superiority of the intensity measurement over the pressure measurement for sound source identification. If the numerical model simulating the radiation of the considered source were perfectly accurate - it would mean that all the boundary conditions would be accurately defined - the identification process using pressure or intensity data would lead to the same accurate results : the actual source magnitude distribution. However, since it is very seldom that the model is completely accurate, one should try to find an optimal approximation of the source radiation. The first part of this study shows that in free field conditions, when the acoustic source is not well represented, the identification of its radiation can lead to better result by using intensity level rather than sound pressure level measurements. The subsequent minimization technique must however be very stable because the minimized functional is not convex. The so called "trust region method" [3] that has been tested for this study seems to be quite efficient for this purpose. The second part of the study deals with the problem of identifying a given sound source when the boundary condition in the model is not well represented. Again, the results illustrate the superiority of the intensity measurement over the complex acoustic pressure measurement. Moreover, it is shown that the trust region minimization technique allows one to establish a good estimate of the power radiated by source elements. The interesting aspect of the identification methods are particularly obvious in the low frequency ranges with reverberant environment when the classical antenna techniques cannot be properly applied. These techniques use simple radiation model (plane wave - spherical wave) that do not take into account all the diffraction phenomena ; they may be quite important in the low frequency ranges. The identification technique that has been applied is based on the use of a finite difference model that takes into account these phenomena.

REFERENCES

[1] F. SANTOSA, "Inverse problems of acoustic and elastic waves", Edition S.I.A.M., Philadelphia, 1984.
[2] B.N. PSHENICHNY et Yu.M. DANILIN, "Numerical methods in extremal problems", Mir publishers Moscow, 1978.
[3] R. FLETCHER, "Practical methods of optimization", Volume 1 et 2, Wiley & sons 1981.
[4] C. LEMARECHAL, "Méthodes numériques en optimisation: développements récents", Compte rendu du séminaire d'analyse numérique, Université Paul-Sabatier (Toulouse III), 1982.
[5] E. LUZZATO, "Méthodes de reconstruction des sources sonores", EDF-DER HE-24/88.100, 1988.
[6] P. MORSE, K. U. INGARD, "Theoretical Acoustics", Mc Graw-Hill Book Cy, 1968.

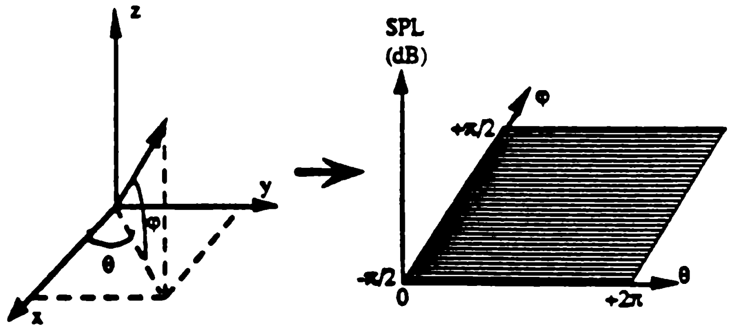


Figure 1 : Representation of the Sound Pressure Levels (SPL) in dB observed on the spherical surface of radius 2λ.

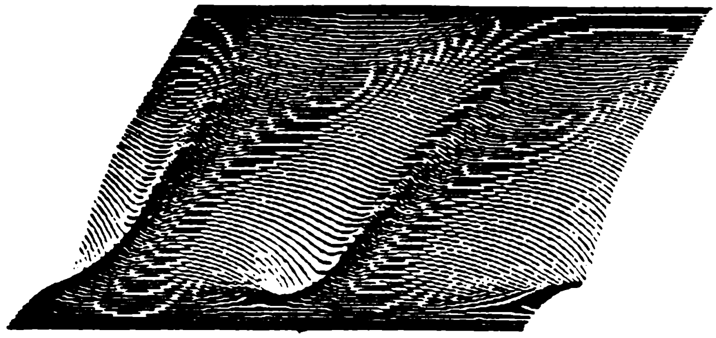


Figure 2 : SPL obtained with the 6 reference sources on the observation spherical surface of radius 2λ.

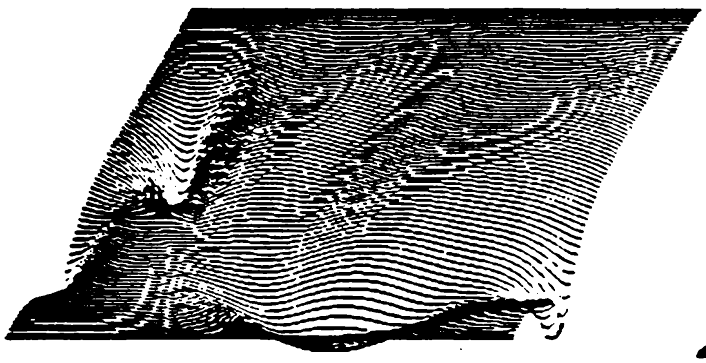


Figure 3 : SPL obtained with 8 identification sources. Pressure modulus identification and minimization of $\phi_p(A)$.

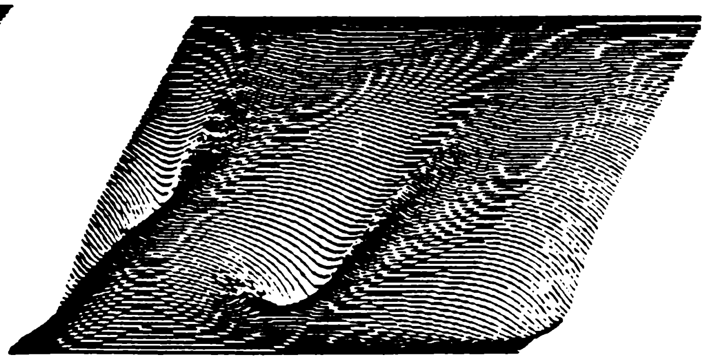


Figure 4 : SPL obtained with 8 identification sources. Active intensity identification and minimization of $\phi_I(A)$.

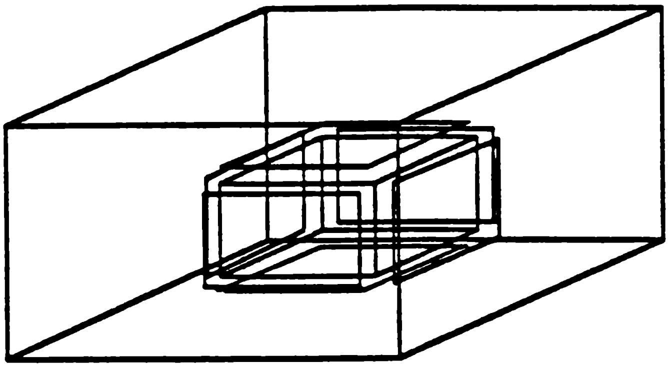


Figure 5 : Geometrical configuration of the acoustic enclosure solved by means of a finite difference technique.

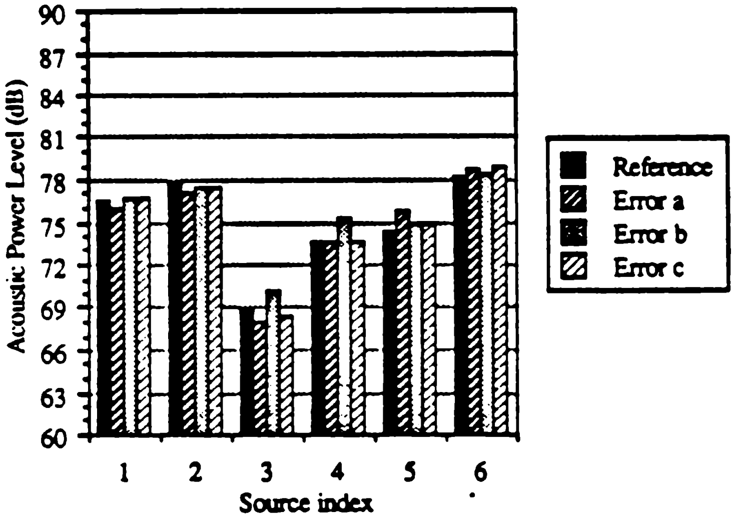


Figure 6 : Acoustic power levels obtained for the 6 surface sources when α = 0.9.

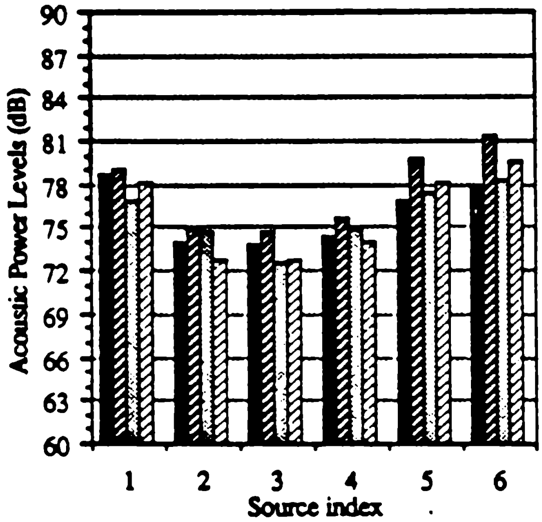


Figure 7 : Acoustic power levels obtained for the 6 surface sources when α = 0.6.

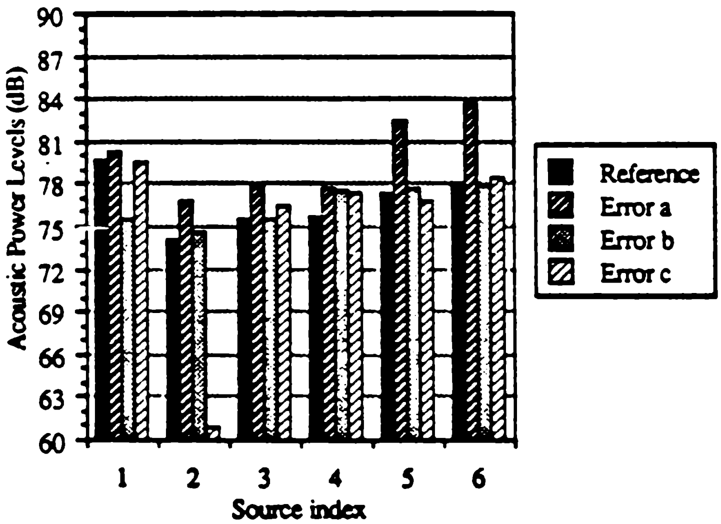


Figure 8 : Acoustic power levels obtained for the 6 surface sources when α = 0.3.

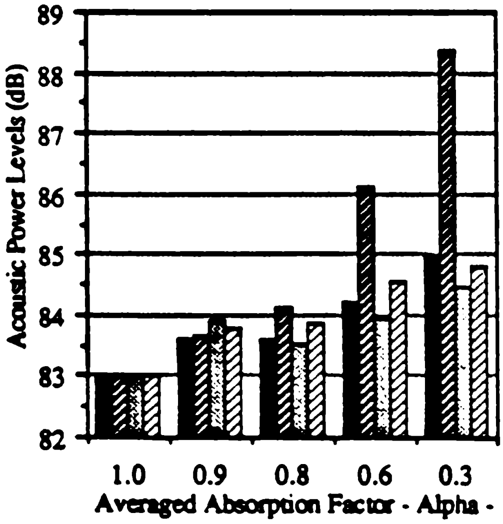


Figure 9 : Total acoustic power levels obtained for the rectangular source with various values of α.