Applications of the Local Energy Approach in elastoacoustic engineering

M. N. Ichchou, A. Le Bot and L. Jezequel
Laboratoire de Tribologie et de Dynamique des Systmes, UMR CNRS 5513
Ecole Centrale de Lyon, 36 Avenue Guy de Collongue, 69130 Ecully Cedex, FRANCE

Abstract

Several engineering problems treated using the **L***ocal* **E***nergy* **A***pproach are presented here. This approach considers a local energy balance, and aims to extend* **SEA** *which is derived from global energy balance considerations. The* **L***ocal* **E***nergy* **A***pproach was successfully used in a number of elastoacoustic engineering problems. For instance, complex and periodic waveguides, complex built up plates, room acoustics, coupled elastoacoustic problems with internal fluid was studied using the local energy considerations. Some numerical and/or experimental results are provided in this review paper to show the interest of this approach. In-house codes based on the* **L***ocal* **E***nergy* **A***pproach are finally presented.*

1 Introduction

The medium and high frequency dynamics is the subject of a fantastic devotion and the great number of works and publications confirm the interest of the scientific community on such a question. In fact, there is still a real need to propose predictive tools well suited to deal with medium and high frequencies and to take into account the numerous features of this domain. Those features make irrelevant the use of the classical methods (finite element, boundary element,...), for many reasons we briefly discuss next.

Among the classical methods which are widely employed in the mastering and prediction of sound and vibration level of industrial machines, let's mention the finite element method as an example. From a global point of view, this method uses a discretisation procedure which is mainly dependant on the complexity of the studied cases and on the frequency band of interest. Indeed, as the frequency increases, the wavelength decreases and a great number of elements are needed. This leads to difficulties in numerical analysis. In addition, in the strict analysis of the features of high frequencies, it should be noticed that the main problem against the use of classical methods is the damping phenomenon which have a leading role. In fact, as the frequency increases, the modes are strongly coupled whilst the approximation of the dynamical behaviour by finite element method advances the superposition of uncoupled modes.

In the light of this brief description, the developpement of further alternatives appears to be needful. Among those alternatives, let's make mention of the well known *Statistical Energy Analysis* (SEA). This method [1] appeared in the early seventies. SEA proposes from a complete energetic mind to analyse the energy transfer between subsystems (set of modes). From an SEA scheme a global tendancy of the total energy is then given. Nevertheless, SEA needs to be improved in order to avoid its deficiencies. The readers can, in this context, bear reference to the review of the *Statistical Energy Analysis* [2] , where an interesting survey and a number of critical comments are given.

Owing to the exposed literature survey, a number of works appeared to enhance SEA robustness and predictivity. Among those tentatives let's report the earlier work of Belov and Rybak [3] and the investigation of Nefske and Sung [4] who proposed the use of the heat analogy conduction to get not only the total energy available in a (SEA) model, but also the space spread of energy density whitin subsystems. This leads to an energy formulation of the dynamical equation of motion instead of the classical displacement based model. This model has been improved by Bernhard and his colleagues [5,6].
Developments done by the authors in the context of energy models started with the so-called general energy method [7,8,9]. The main goal of this method was to reformulate the classical displacement models using four energy variables: the total energy as well as the lagrangien energy density, the active and the reactive energy flow. Using those energy parameters, "exact" dynamical behaviour for simple canonical one-dimensional structure (bars and beams). This method fails however in representing complex one-dimensional and multi-dimensional and does not present any numerical advantage in comparison with the classical displacement models. However, from the elimination of the lagrangien energy density and the reactive energy flow, this general energy formulation leads to an interesting energy model well suited for medium and high frequency dynamics. In fact, from a propagative approach, it has been shown that the lagrangien energy density and the reactive energy flow is mainly linked to wave interferences or singularities. So that, taking into account only the uncoherent contribution of waves to the energy variables leads to the formulation of the **L**ocal **E**nergy **A**pproach concerned here. References [7-24] give in depth the formulations used in the context of **L**ocal **E**nergy **A**pproach applications. The main goal of this paper is to synthetise those applications in order to show the interest of this appraoch in the context of elastoacoustic engineering.

2 Structural Dynamics

The first kind of applications concerns pure structural dynamics tests. The **L**ocal **E**nergy **A**pproach was first analytically validated for very simple one-dimensional (mainly rods and beams). It has been then extended in order to deal with complex one-dimensional (beamlike structures, truss). The **L**ocal **E**nergy is a wave based **A**pproach. A numerical hybridation with finite element codes was developped and implemented in order to provide wave characteristics. This hybrid method provides diffusion matrix for complex interfaces as well as power input for non typical thin walled strutures. Some examples are presented here. Finally, the analysis of vibrating plates is proposed. Numerical and experimental comparisons show the validity of the local approach.

2.1 Complex one-dimensional

Let us consider a complex numerical test in order to validate the computed local energy approach. The concerned structure is extracted from a submarine construction[1] and is principally constituted from wave guides propagating traction compression and flexural waves. Figure 1 presents the studied structure. It is a plane truss composed from twenty four wave guides. Compressional and flexural waves have been considered in the energy computation. The source is assumed to be a punctual force of vertical or horizontal kind. This example offers a number of difficulties in terms of the energy method use. In fact, a number of interface of 2, ... 6

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[1] Provided by CERDAN DCN

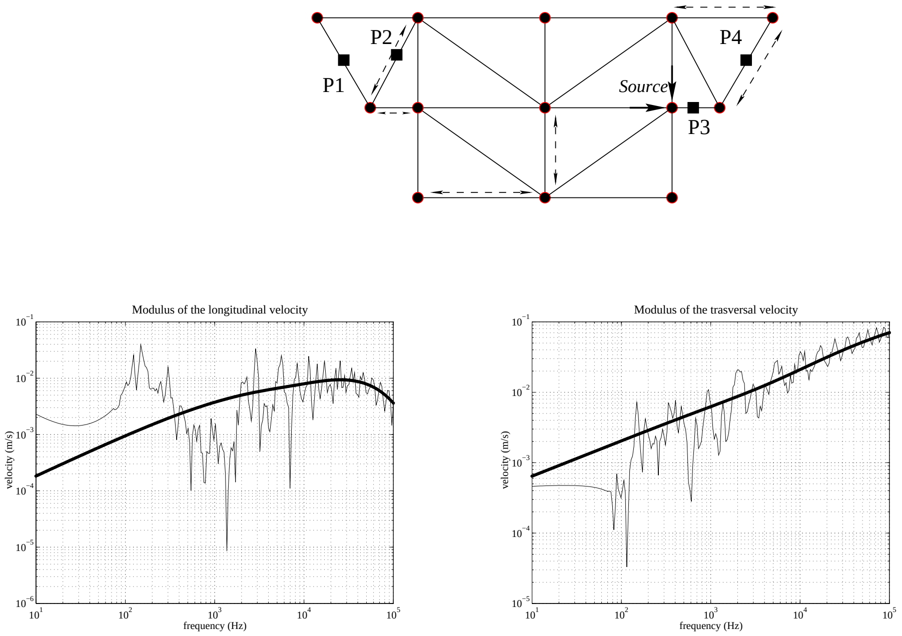

Figure 1: LEA predictions in comparison with an exact calculation: :upper figure shows the truss under study. Lower figures give the extensional (P3) and transversal (P3) velocities (tick line) LEA predictions (normal line) exact calculations

components kind and an excitation on a five component interface is introduced. As only, two waves are dealt with here, an energy model of 96 energetic degrees of freedom is needed. The diffusion matrix of the most general connection form have been computed, tested for some cases and thus applied in its general form. Figure 1 gives an outline of energy computation results. In fact, the local energy results are a direct prediction of the mean values of expected dynamical levels. The latter, indeed, smooths the response and gives with a low computational cost an averaged prediction of the expected results.

2.2 Periodic one-dimensional

The structure studied as an illustrative example (Figure 2) has been widely used in the literature concerning periodic structures. It consists of a finite configuration of n span described by an Euler Bernoulli beam simply supported at their connections and with a slope flexibility. Mead [25] used this example in order to illustrate the transfert matrix treatment of periodic systems. In view of the construction and validation of the local energy model, the dispersion law corresponding to the studied case are first computed, here, the expression obtained by Mead [25] is directly used.

Figure 2 gives the dispersion curve of the periodic beam, this figure shows clearly the well-known periodic system features concerning frequency pass and blocked zones. The local energetics given below take into account this properties, as it describes the non coherent energy propagation. Results presented in Figure 2 are representing the frequency behaviour of the kinetic energy at a given position in a given span showing clearly the interest of the local energy approach. In fact, the reference results coming from the resolution of the classical equation of motion, has an oscillating aspect with a drastic energy failure at the local of the blocked frequencies. The prediction of the local energy approach agrees very well with the reference results, as it gives precisely a mean energetic value in the pass frequency band, and it follows the non oscillating behaviour of the reference results in the blocked frequency band.

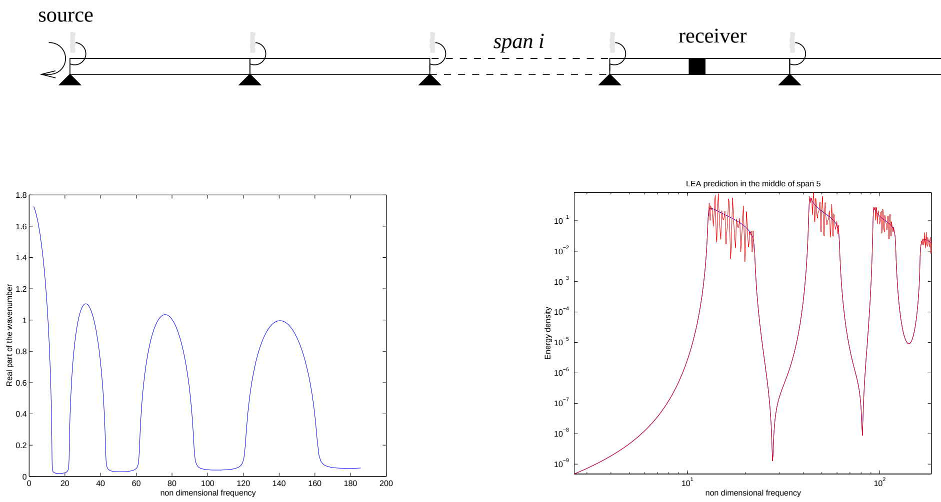

Figure 2: LEA predictions in comparison with an exact calculation : upper figure presents the Mead periodic beam excited by a moment. Lower figures give the dispersion curve and a comparison between energy computations at the receiver location

2.3 Thin walled structures

The outline of the procedure used in order to deal with thin walled structures or mostly elastoacoustic complex waveguides is exposed here. It is based upon a hybridation of the **L**ocal **E**nergy **A**pproach with existing finite element codes. Up to now, an in-house code and an interface under the OPTIMA[2]/NASTRAN environment are running. This hybridation allows the propagation constants to be numerically estimated. The interface uses existing finite element codes abilities. Hencefore, from reuses of a finite element model, dispersion curves of realistic thin walled structures or elastoacoustic waveguides are extracted by means of a convenient post-treatment of the finite element model.

The example shown here considers a canonical hollow structure of rectangular section (Figure 3). The proposed results comes from OPTIMA validation. Results show the wave structural complexity as the frequency increases. The provided wave characteristics are used as input data for the **L**ocal **E**nergy **A**pproach. It should be noticed that those wave information inputs can also be used in the context of SEA for modal densities as well as power input determination.

Figures 4 clear up the interest of the **L**ocal **E**nergy **A**pproach in the case of thin walled treatment. It illustrates comparisons between measurement and local energy prediction.

2.4 Built-up plates and shells

Built-up plates is considered in this subsection. Two examples are briefly presented. The first example, is a simple mock-up of a car roof[3]. The second example considers a curved shell, which is approximated by a set of assembled plate. Both examples were experimentally checked. The source is of mechanical kind, an impedance head was used in order to measure the injected power. The mock-up presented in Figure 5 represents a reduced scale roof of an automotive.

Figures 5 and 6 present some results from the first built-up plate structure and the shell tested case. In both cases, the energy density versus frequency is given in some locations of the considered mock-up. Comparisons are made between the measured energy density, readily extracted from the point velocity measurement, and the prediction of the **L**ocal **E**nergy

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[2] Peugeot Citroen platform

[3] under PSA contract

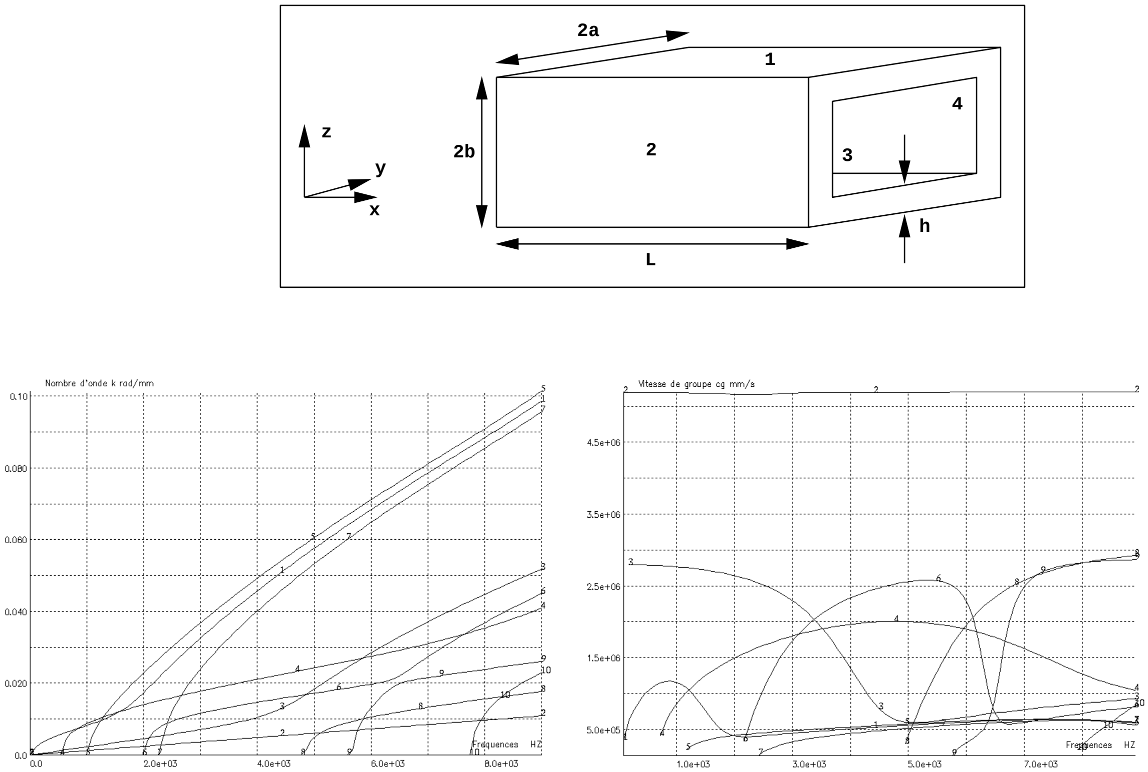


Figure 3: wave characteristics by means of an hybrid finite element code. Upper figure shows the canonical tested structure. Lower figures give the numerical wavenumbers as well as the wave velocities computations

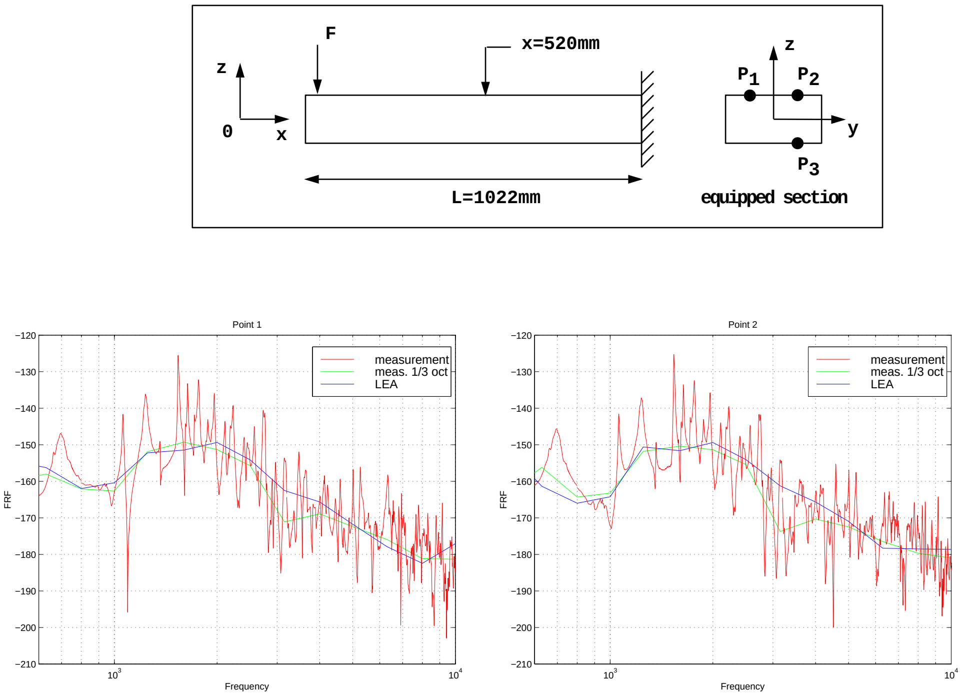


Figure 4: LEA predictions in comparison with experimental results : Upper figure shows the tested structure. The lowers give transverse velocity measurement and energy predictions in point 1 and point 2.

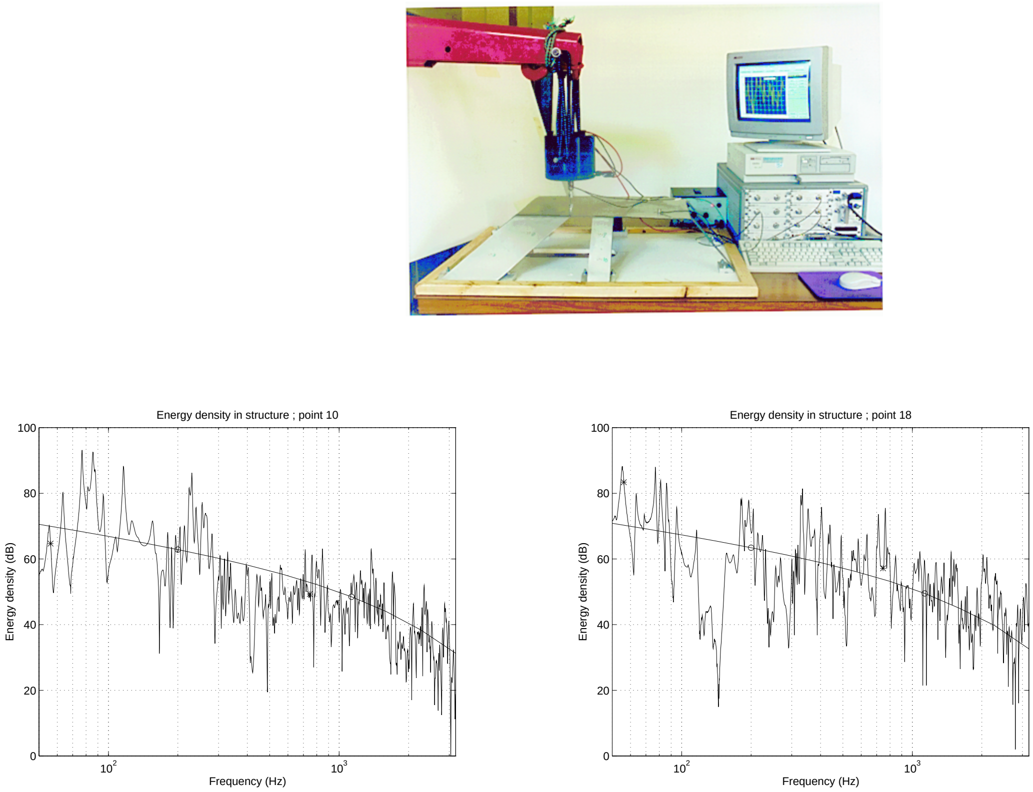


Figure 5: LEA predictions in comparison with experimental results : Upper figure show the mock-up of the car roof constituted from 7 plates with the experimental set-up . The lower figures give comparisons between (o) local energy predictions and (*) measurements.

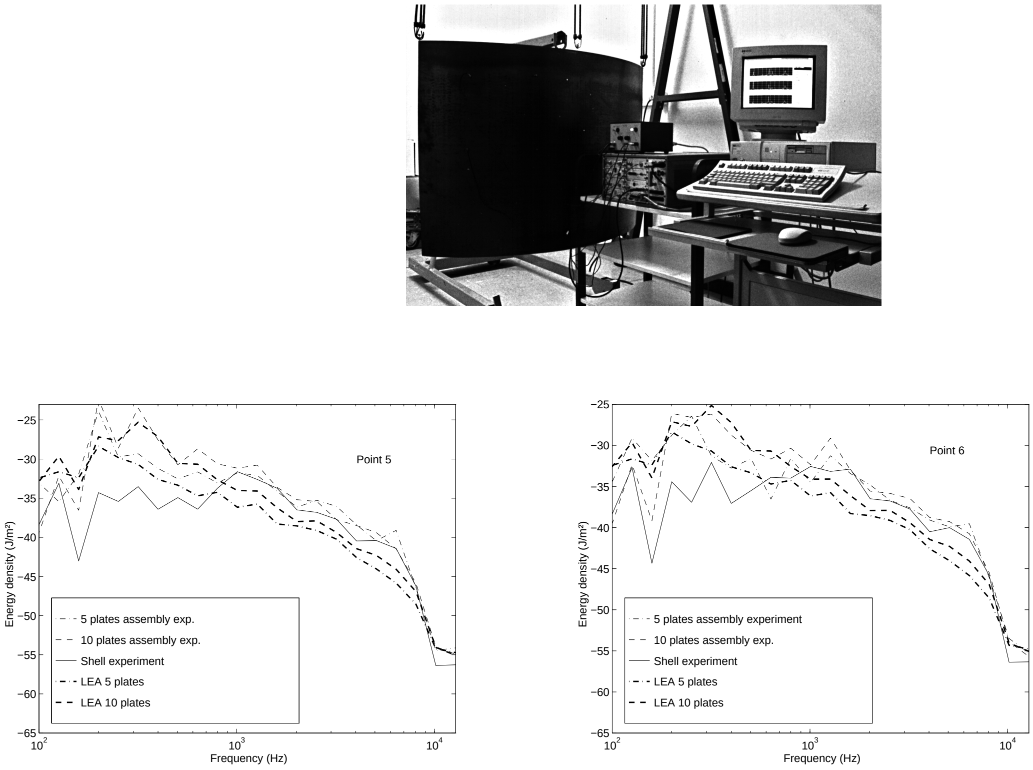
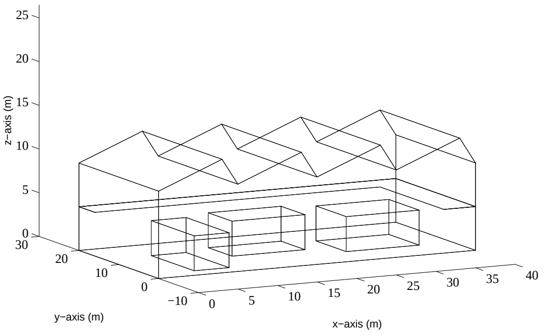


Figure 6: LEA predictions in comparison with experimental results : Upper figure show the shell under interest with the experimental set-up . The lower figures give comparisons between local energy predictions and measurements.



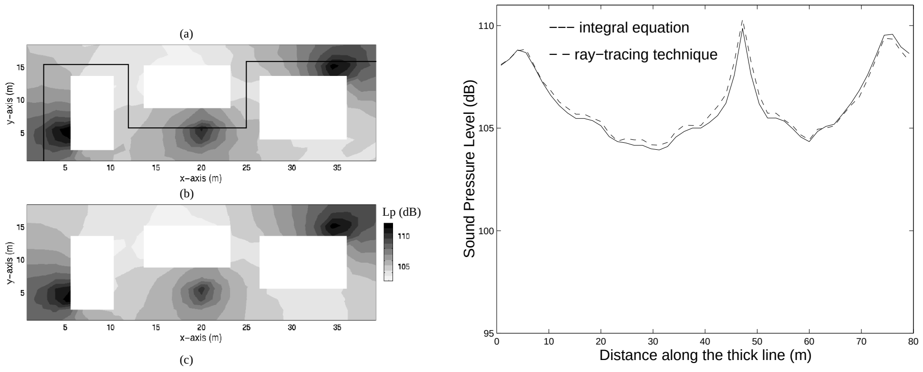


Figure 7: Noise in a factory with three noise sources: comparison of Sound Pressure Level (Lp) by (a) the RAYON software and by (b) the CeReS software on a horizontal receiver plane 2m above the floor. (c): Direct comparison of SPL along the thick line.

Approach. An equivalent damping ratio was estimated for each third octave band. The results and comparisons shown here confirm that the **L**ocal **E**nergy **A**pproach prediction well agree with the frequency average of the experimental measurement.

3 Room Acoustics

Rooms acoustics validation is in concern in this section. The **L**ocal **E**nergy **A**pproach was compared with the well known ray-tracing technique. An integral equation describing the local energy formulation was solved. An appropriate numerical scheme using the collocation method is adopted to solve a Fredholm equation on energy. The second comparison provided in this section concerns a typical heating or ventillation problem. It concerns here a coupling between an acoustic duct and an acoustic cavity. The acoustic duct propagation modes was first identified. Experimental results are given.

3.1 Room acoustics

In the case of room acoustics, results of the **L**ocal **E**nergy **A**pproach has been compared with results;of the well-known ray-tracing technique. The former method is involved in a specific in-house software (CeReS) designed for the resolution of a Fredholm equation on energy with the collocation method. The acoustical enclosure may be of various shape and size with absorbing

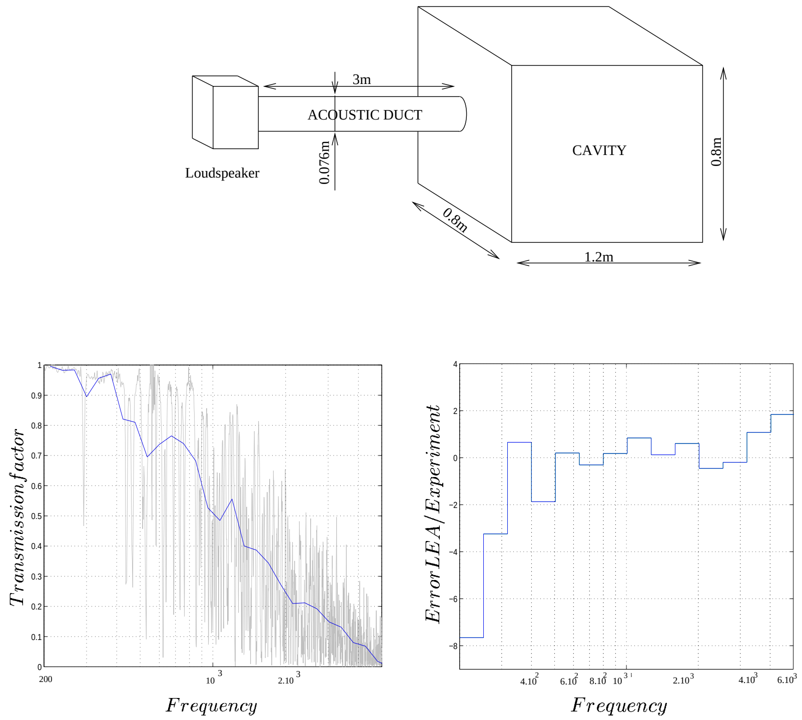


Figure 8: LEA predictions in comparison with experimental results : Upper figure shows the duct-cavity experimental test. Lower figures gives first the transmission efficiency at the duct/cavity interface versus frequency, and the difference between the LEA predictions and the experimental results.

walls. The latter method is implemented into the software RAYON2.0[4]. In Figure 7, results of calculation of both methods are compared for the particular case of a virtual factory with some objects inside. Comparisons between both compuations agree adequately.

3.2 Acoustic ducts - rooms acoustics coupling

The sound transfer through air ducts and its interaction with rooms and cavities is a typical acoustical center of interest. For instance, prediction and control of unwanted sound in heating and ventillation systems is an important task to deal with. For this kind of problems, one is often interested in the energy propagation and transfer prediction rather than a complete charaterisation of the solutions by means of modal decomposition or other predictive methods. In addition, further uncertainties make modal behaviour irrelevant and inadequate. Such phenomenon is accentuated in high frequency domain. Using a propagative decomposition of the response, energetics of acoustics in ducts with absorbing conditions was considered (Figure 8). A particular treatment of energy transfer at boundaries and radiated from the ducts was operated, leading to a relevant prediction of sound propagation in a large frequency domain.

4 Coupled Elastoacoustic problems

Coupled elastoacoustic problems are finally presented in this section. The first exemple chosen here, concerns fluid structure coupling. In fact, coupling between internal acoustic field with vibrating shell is considered. Pipes filled with fluid is used here and some illustrations

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[4] EDF/DER software

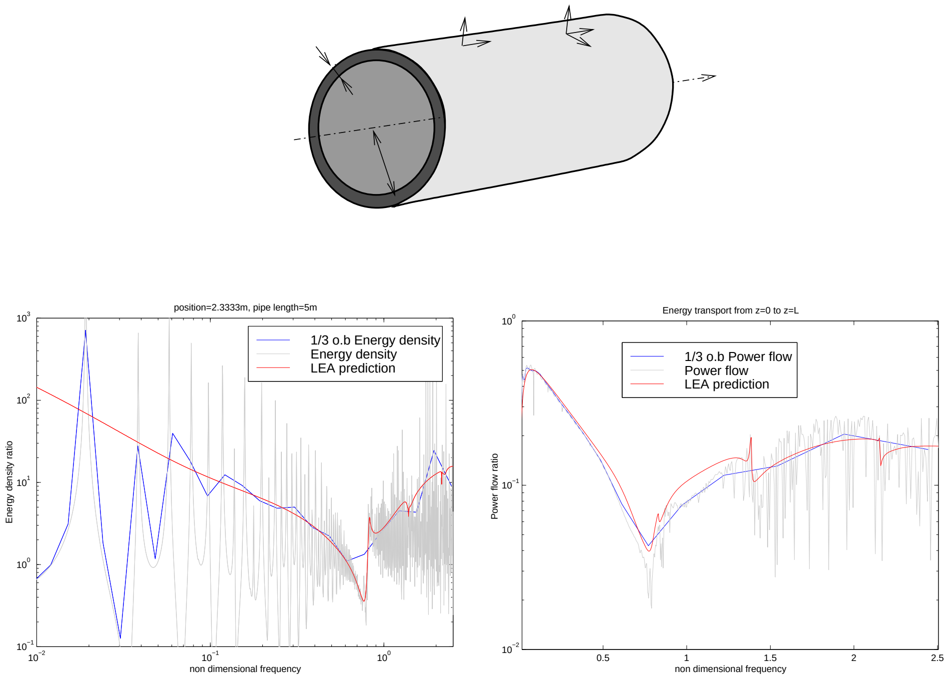

Figure 9: LEA predictions in comparison with an exact calculation : upper figure shows the simple fluid filled pipe considered here. The lower figures give comparisons of LEA computations and 1/3 octave averaged exact results

given. Eventually, radiating of plates is considered allowing the prediction from a **L**ocal **E**nergy **A**pproach of the sound radiated in both subsonic and supersonic domains.

4.1 Fluid structure coupling

Fluid filled pipes are often used in manufacturing products. Up to now, the high frequency analysis of such structures by means of SEA or alternatives remains an important issue and a susbject of research and developments, because of their intrinsic dynamics leading to fluid structure coupling effects. From a state space formulation of the problem, the propagative behaviour of such systems was obtained. The propagative content leaded to a relevant representation of energy exchanges between propagative modes and to the energy spread within the systems. Comparative results are given in Figure 9 in order to validate the **L**ocal **E**nergy **A**pproach model. The case considered here, is a pipe line filled with water. More details are given in reference [21].

4.2 Radiation of planes structures

Radiation of planes structures was considered, in the context of **L**ocal **E**nergy **A**pproach. Both the subsonic and the supersonic cases were studied. The energy transfer and considerations were taken into account. Notably, the edge radiating effect was introduced. Figure 10 shows a simple unbaffled vibrating plate radiating in free space. The plate was excited by an electrodynamic shaker. The velocity as well as sound pressure level was evaluated experimentally. The results of the comparison between the **L**ocal **E**nergy **A**pproach predictions and measurement are given in Figure 10. Additional results are given in [22].

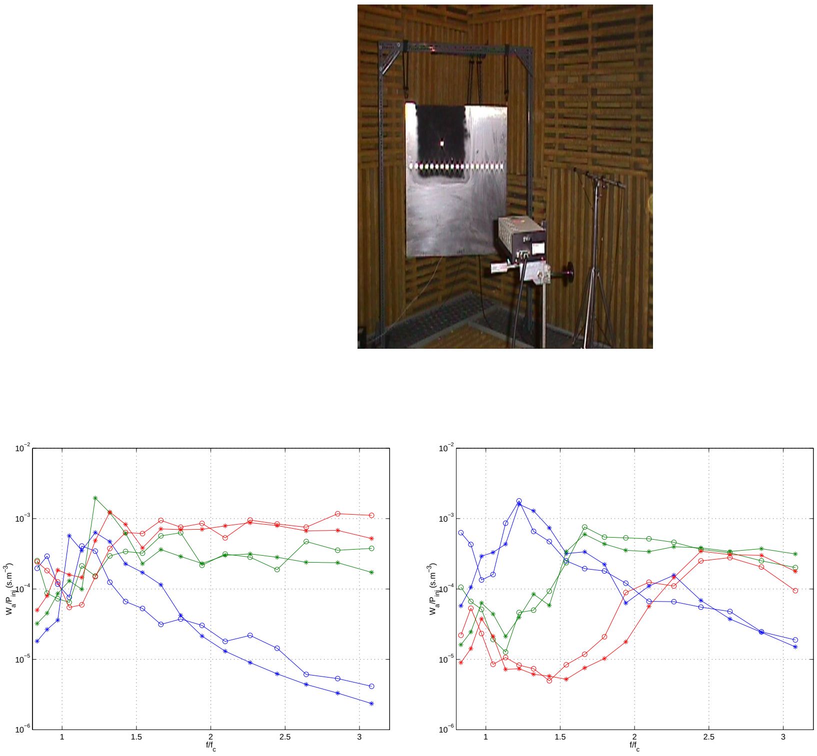

Figure 10: LEA predictions in comparison with experimental results : Upper figure shows a simple plate radiating in an anechoic chamber. Lower figures give comparisons of the acoustic energy in some points of the chamber (o) experiments and (*) LEA calculations in the supersonic case

5 Implementation of Local Energy Approach and future works

Up to now several in-house codes based on the **L**ocal **E**nergy **A**pproach formulations are running. Improvments and extensions are also in progress. In fact, thin walled structure treatment for example is conducted under an interface with the finite element code NASTRAN allowing computation of wave characteristics for complex structure. An integral formulation of the **L**ocal **E**nergy **A**pproach is computed in a software called CeReS permitting **L**ocal **E**nergy **A**pproach study of plate assembly as well as room acoustics. Evantually, a software called CAT/LEA is running. It solves a modified differential energy formulation of the **L**ocal **E**nergy **A**pproach and is implemented under the commercial CATIA/ELFINI software. This CAT/LEA software uses all the CATIA abilities and allows **L**ocal **E**nergy **A**pproach vibration and noise prediction of complex products. Figure 11 gives an illustration of the CAT/LEA computationnal results.

The **L**ocal **E**nergy **A**pproach presented in this applications review paper is still under study and improvments. In fact, intensive efforts are made in view of extending this approach to the elastoacoustic field. In particular, recent developpments permitted the formulation of a **T**ransient **L**ocal **E**nergy **A**pproach [24]. In addition, from a numerical point of view, further works are also conducted in view of the hybridation of this approach with finite element or boundary element codes. Evantually, from experimental question, specific identification and updating techniques will be studied in order to provide tools needed in the **L**ocal **E**nergy **A**pproach modelling.

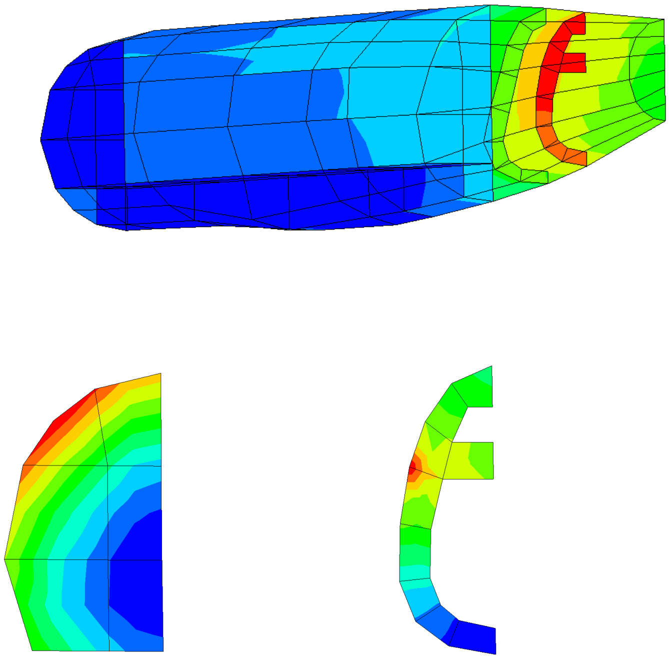

Figure 11: CAT/LEA software[6] : Upper figure shows an aerospace typical component with a point source. Lower figures give LEA energy spread predictions at two distinguish region of the fuselage

6 Acknowledgments

The authors gratefully acknowledge Mr R. Aquilina (CERDAN, Toulon), Mr E. Luzzato (EDF, Paris), Mr E. Garrigues and E. Nicot (DASSAULT, StCloud) and Mr B. Hazet (PSA, Velizy) for their finantial aid, their scientific collaboration, advices and encouragements. The authors gratefully acknowledge inputs and figures provided by P. Moron, A. Bocquillet, L. Houillon and V. Cotoni.

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